LSA 98
Lightweight Structures in Architecture,
Engineering and Construction
IASS/IEAust/LSAA International Congress
5-9 October 1998
Sydney, Australia

Structural Optimization and Form Finding of Lightweight Structures

Kai-Uwe Bletzinger* and Ekkehard Ramm‡

*Institut für Baustatik
University of Karlsruhe
Kaiserstr. 12, D-76131 Karlsruhe, Germany
Tel.: +49 721 608 2283
Fax: +49 721 608 6015
Email: kub@bau-verm.uni-karlsruhe.de

‡Institute for Structural Mechanics
University of Stuttgart
D-70550 Stuttgart, Germany
Tel.: +49 711 685 6123
Fax: +49 711 685 6130
Email: eramm@statik.uni-stuttgart.de

1 Introduction

Any design is driven by a combination of different objectives and constraints such as for example function, cost, aesthetics, manufacturing and other technical requirements. The structural component is mainly influenced by the laws of mechanics and becomes dominating for lightweight structures. A lightweight structure is defined by the optimal use of material to carry external loads or pre-stress. Material is used optimally within a structural member if the member is subjected to membrane forces rather than bending. The objective of an optimization procedure to determine layout and shape of a lightweight structure is, therefore, to minimize bending or more general, to minimize the strain energy rather than structural weight as the term *lightweight* may imply.

As a consequence, typical structural principles have evolved depending on the kind of membrane action: pre-stressed cables and membranes for tension structures, and arches and shells for structures in compression.

The principle of lightweight structures is well known since centuries and leads to very successful experimental methods as for example the inverse principles of hanging models to determine the shape of compression structures [1] or the soap film analogy to determine the shape of pre-stressed membranes and tents [2]. Bending is omitted a priori by the use of proper material or structural members as e.g. chains or cloths. The experimental procedures have been simulated by equivalent computational methods [3]. Hanging models are described by conventional non-linear finite element analysis of elastic bodies with respect to large displacements and small deformations. The deformed shape of the experimental specimen is defined as the shape of the optimal undeformed structure. The singular mathematical properties of the soap film experiment [4], however, implied the development of several concurrent methods, as e.g. dynamic relaxation [5], the force density method [6], the updated reference strategy [7] or methods of modified linearization [8,9].

On the other hand, minimizing bending means minimizing the strain energy. This criterion is implicitly fulfilled by the experimental methods and their simulation equivalents. The methods of structural optimization use it as an explicitly formulated objective function within a rigorous mathematical framework [10-12]. This approach is very general with respect to the choice of variables and to additional constraints and objectives, as for example structural buckling. Furthermore, special methods of structural optimization deal with the optimization of structural topology which is far beyond the scope of experiments and their computational simulation [13-15].

We state that three different lines of computational form finding methods exist: (i) simulation of hanging models, (ii) simulation of soap films, and (iii) structural optimization again divided into shape and topology optimization. The aim of the present paper is to show their differences and common aspects when applied to the special task of maximizing stiffness.

2 Maximum stiffness - minimal strain energy

In a structural sense one gets the most out of the used material if the structure is the stiffest possible alternative. However, maximizing the stiffness without restricting the material is meaningless. Obviously the total stiffness can be improved if more and more additional material is built in. In particular for lightweight structures this does not make sense. The term ‘maximum stiffness’ must always be understood relative to the mass of the structure. Consequently, a structure subjected only to its self-weight can never be optimal. Otherwise it would not exist. Every optimization strategy would tell you this result, ruthless to the designer’s intentions.

The duality of maximum stiffness with respect to constant mass or minimum weight with respect to a prescribed stiffness is known since hundred years by the work of Maxwell at the end of the last century and Michell from 1904 [16]. Michell’s work is of high theoretical interest. It deals with the optimal distribution of mass in space and results in a quasi-continuous system of truss-like structures where the members follow the trajectory lines of constant strain. Although many of these structures are kinematical and practically useless because they are optimized only with respect to one load case, the basic principles can be observed in many technical and natural truss-like structures. The ideas of Maxwell and Michell are also the basis for the development of very successful and nowadays very popular optimization techniques which deal with the optimization of structural topologies, i.e. the optimal layout of trusses and frames in two and three dimensions.

Maximizing the stiffness of a structure means optimal rearrangement of material in space starting with an initial structure. We talk of ‘shape optimization’ if the material density is constant, i.e. the structural contour or the spatial extension is modified. If the material density is optimized we deal with ‘topology optimization’. In this context structural holes are defined by regions where the material density is zero and vanishes.

A few words are added on the variables. In the context of structural optimization, we distinguish between two sets of variables: (i) the design variables $\boldsymbol{s}$ and (ii) the state variables $\boldsymbol{u}$. The state variables describe the mechanical deformation due to given loads and material properties. In standard finite element methods $\boldsymbol{u}$ are usually the structural displacements, discretized at the finite element nodes. The design variables $\boldsymbol{s}$ describe the structural layout. In shape optimization this is the geometry of the shape, in topology optimization it is the material distribution and density.

In the context of structural optimization the maximization of stiffness means the minimization of strain energy with respect to the design variables $\boldsymbol{s}$. On the other hand, the minimization of the potential energy with respect to the state variables $\boldsymbol{u}$ states the equilibrium of the structure. Equilibrium is a structural constraint which must always be fulfilled. Since the strain energy is as well a part of the objective function as of the constraint we can state some special properties of the problem concerning the sensitivity analysis, i.e. the determination of the derivatives of the strain energy with respect to the design variables.

We define π as the potential energy consisting of the strain energy of the internal stresses and strains and the external energy π_e of the applied load and displacements. Since the mass constraint is independent of $\boldsymbol{u}$ we can omit it for the following argument. Now, the optimization problem states as:

$$\min_{\mathbf{s}} \pi_i(\mathbf{s},\mathbf{u}) = \frac{1}{2}\int_V \sigma\varepsilon \, dV; \quad s.t. \; \delta_{\mathbf{u}} \pi(\mathbf{s},\mathbf{u}) = \delta\pi_i + \delta\pi_e = 0 \tag{1}$$

Assuming linear elastic structural behavior the problem is rewritten in terms of the discrete nodal displacements $\boldsymbol{u}$, the stiffness matrix $\boldsymbol{K}$ and the load vector $\boldsymbol{R}$:

$$\begin{aligned} \pi_i &= \frac{1}{2}\mathbf{u}^T\mathbf{K}\mathbf{u} \;\rightarrow\; \min_{\mathbf{s}} \\ s.t.\delta \quad \pi &= \mathbf{K}\mathbf{u} - \mathbf{R} = 0 \end{aligned} \tag{2}$$

From the equilibrium constraint we determine the displacements and their derivatives with respect to the design variables $\boldsymbol{s}$:

$$\begin{aligned} \mathbf{u} &= \mathbf{K}^{-1}\mathbf{R} \\ \mathbf{u},_{\mathbf{s}} &= \mathbf{K}^{-1}\left(\mathbf{R},_{\mathbf{s}} - \mathbf{K},_{\mathbf{s}}\,\mathbf{u}\right) \end{aligned} \tag{3}$$

Then, the total derivatives of the objective with respect to $\boldsymbol{s}$ are:

$$\begin{aligned} \frac{d\pi_i}{d\mathbf{s}} &= \frac{1}{2}\mathbf{u}^T\mathbf{K},_{\mathbf{s}}\,\mathbf{u} + \mathbf{u}^T\mathbf{K}\,\mathbf{u},_s \\ &= \frac{1}{2}\mathbf{u}^T\mathbf{K},_{\mathbf{s}}\,\mathbf{u} + \mathbf{u}^T\mathbf{K}\,\mathbf{K}^{-1}\left(\mathbf{R},_{\mathbf{s}} - \mathbf{K},_{\mathbf{s}}\,\mathbf{u}\right) \\ &= -\frac{1}{2}\mathbf{u}^T\mathbf{K},_{\mathbf{s}}\,\mathbf{u} + \mathbf{u}^T\mathbf{R},_{\mathbf{s}} = -\pi,_{\mathbf{s}} = -\frac{\partial\pi}{\partial\mathbf{s}} \end{aligned} \tag{4}$$

It turns out that the total derivative of the strain energy is equivalent to the negative partial derivative of the potential energy with respect to $\boldsymbol{s}$. This means, that for this special design objective the very time consuming determination of $\boldsymbol{u}_{,s}$ (response sensitivity analysis) is not necessary which usually dominates the optimization process. A very efficient algorithm can be developed which alternates between one step of structural analysis and one step of optimization to determine the next structural improvement.

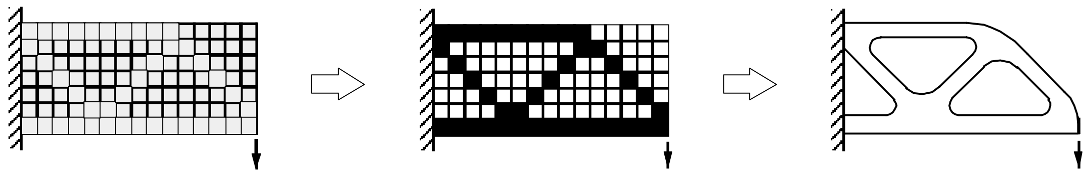
3 Optimization of structural topology; Table 1

As already stated the following relies on the work by Maxwell and Michell. For further information refer e.g. to Maute, Bendsøe, Rozvany [13-15].

Loosely speaking, topology optimization is to decide where to put material in space and where not and how the evolving contours have to be shaped. This ‘material’ approach to topology optimization starts with the definition of a design space in which the resulting structure will be imbedded. The structural layout is defined by the material distribution $\chi(\boldsymbol{x})$ as a function of the location $\boldsymbol{x}$ in the design space which indicates the existence of material, Fig. 1:

$$\chi(\boldsymbol{x}) = \begin{cases} 0 \rightarrow no\ material \\ 1 \rightarrow material \end{cases} \tag{5}$$

Fig. 1: Topology optimization



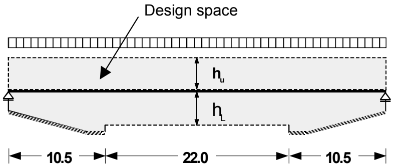
Usually, the discretization of χ is assumed to be identical to the finite element mesh which is used to discretize the design space. The parameters χ_i of the finite elements are the design variables $\boldsymbol{s}$ of the topology optimization problem. A key point is the material model which defines the relations between the material tensor, the material density and the material distribution. The most simple alternative is to fill or not to fill a finite element in the design space with isotropic linear elastic material and to maximize the stiffness. The result would be a very rough approximation of a Michell structure. To get closer to the quasi-continuous nature of Michell structures more precise material models have to be used. It is most important to model the directional properties of anisotropic material. Indeed, there exist an enormous amount of different approaches which lead to more or less discrete Michell structures. Many of them start with a microscopic model and apply homogenization methods with deep roots in material science. In all cases the objective function is strain energy. Of course, the total mass in the design space is prescribed by a fixed value.

3.1 Case study - Bridge design by topology optimization (Table 1; Ploch [17])

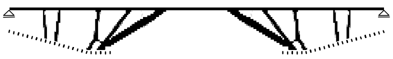
Topology optimization is applied to determine the layout of a bridge considering the diverse design spaces, boundary conditions, load cases and amount of given mass. The design space is

Table 1: Conceptual Bridge Design by Topology Optimization (J. Ploch)

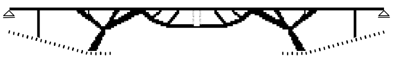
The aim is to determine the influence of various design parameters on the optimal topology layout.



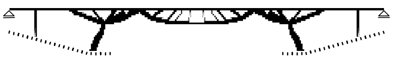
Left:
Upper and lower dimensions of design space, positions of left and right support, embankment where additional supports may be located, and the main load case.



h_L = 4.2 m; h_U = 0.0 m



h_L = 2.2 m; h_U = 0.0 m



h_L = 1.6 m; h_U = 0.0 m

h_L = 1.0 m; h_U = 0.0 m

h_L = 1.0 m; h_U = 1.4 m

h_L = 1.4 m; h_U = 2.8 m

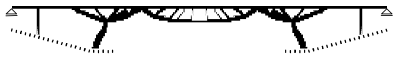
Left:
variation of design space

- uniform vertical load
- potential fixed support at embankment
- same mass for each design

Right:
variation of support

- uniform vertical load
- fixed design space
  h_L = 1.6 m; h_U = 0.0 m
- same mass for each design

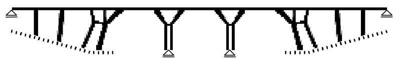
embankment: fixed

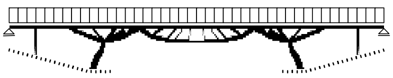


vertically supported

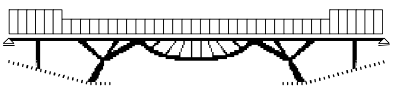
partially vertically supported

Fixed and additional supports

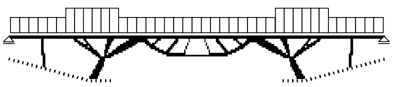




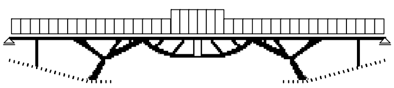
reference load case



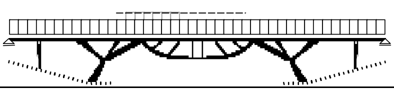
load case 1



load case 2



load case 3



multiple load case
cases 1 to 3

Left:
variation of load

- fixed design space
  h_L = 1.6 m; h_U = 0.0 m
- embankment: fixed supports
- same mass for each design

chosen according to the structural situation of Maillart's bridge at Liesberg, Switzerland, from 1935. For each setting the material distribution is optimized with respect to stiffness for given mass based on a macroscopic orthotropic material model. Linear elastic structural response is assumed.

In a first step the influence of the size of the design space is investigated considering only one load case, i.e. a uniform vertical load along the given roadway layer. Potential boundary conditions of the emerging structure at the embankment are modeled as fixed supports. All alternatives are of the same mass. The different results indicate that, if possible, it is preferred to transmit the forces as directly as possible to the ground or to generate an arch.

Further parameters had been the support conditions, different load cases, also in combination, and, the amount of available material. The optimization results show clearly that the uniform load is dominant for optimum layout. Based on the maximization of stiffness topology optimization provides a powerful and robust tool to investigate easily fundamental questions about the structural design and to generate conceptual design ideas. Of course, further investigations of the structural behavior are necessary which are not considered by the optimization process.

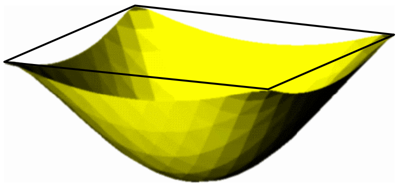
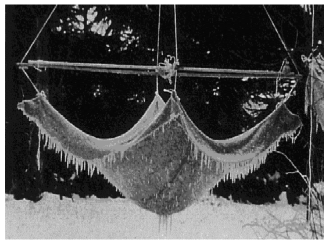
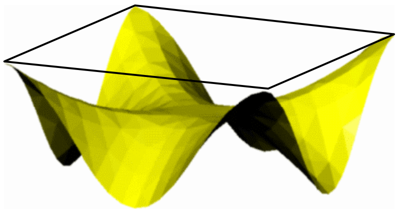
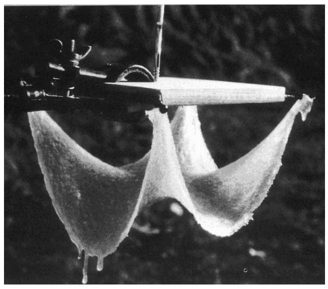
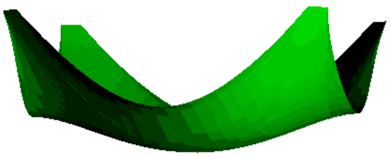
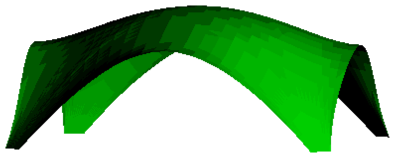
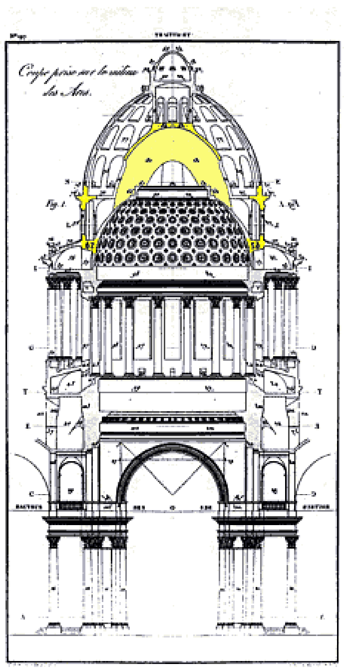
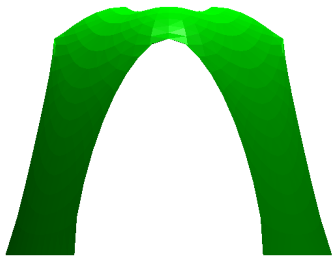
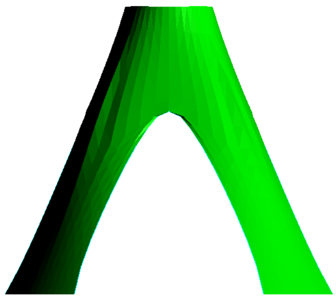
4 Hanging models - inverse methods I; Table 2

The hanging chain and its inverse is one of the oldest methods which are known to generate the shape of an arch which is free of bending, subjected only to compressive axial force. The method has been used intensively during the centuries, e.g. by Antoni Gaudí, to give one well known name among all the others. Extended to two directions to define the shape of shells the hanging model concept has been brought to perfection by Heinz Isler [1].

If we trace the procedure of minimizing strain energy of a structure originally subjected to bending we realize that first the shape is modified to reduce bending which is energetically very inefficient. In the following steps the best of the alternatives acting in a pure membrane stage is searched for. Looking at hanging models from the optimization point of view the bending reduction is implicitly fulfilled just by taking a structure which is not able to resist bending and shear, e.g. the chain or textile cloths in two directions. The goal of hanging models is to perform the transition from a 'bending structure' to a 'membrane structure'. A further systematical search for the optimal structure among the class of 'membrane structures' is not performed.

The optimal shape generated by hanging models is the result of a mechanical deformation. It is dominated by the size of the undeformed original piece of material (chain, cloth) which has been used for the experiment. In two dimensions for the generation of shells the choice of the initial shape is also critical with respect to wrinkles and folds which may develop during the deformation. The orientation of material fibers of anisotropic or woven material with reduced in-plane shear resistance may totally change the result. The example at the top of Table 2 shows even a change from a positive to a negative curved surface. The implicit interaction of initial and optimal shape through the mechanical deformation yields that the design variables are not at all obvious. It might be very complex to identify and to vary them. The variety of possible solutions is almost infinite, because a further classification of structural quality besides the absence of bending is not part of the method. Finally, stability effects cannot be considered by hanging models.

Table 2: Hanging models

The influence of cutting pattern and fibre orientation	Numerical simulation	Ice experiments by Heinz Isler
Data of numerical simulation: Intially plane plate, distributed vertical load. Supported at the vertices. Anisotropic material, fibre orientation 0°/90°. Ratio strain energy / load: 4.98		
Intially plane plate, distributed vertical load. Supported at the vertices. Anisotropic material, fibre orientation ±45°. Ratio strain energy / load: 2.90 Stiffer than model above.		
	Generation of shell geometries < Hanging model and its inverse. >	
	Left: St. Geneviève, the French Pantheon in Paris (1757-1790). Jean-Baptiste Rondelet used the rotated shape of a hanging chain subjected to distributed load to define the shape of the middle dome. However, he did not consider the holes in the dome and the weight of the heavy latern. Right top: Numerical form finding of middle dome by a hanging membrane. Distributed area load ('snow'). Note the wrinkles at the top. Right bottom: Shape due to ring load approximating lartern load. Note the buckles in the flank.	 

The numerical simulation of hanging models by finite element methods is a standard task of non-linear analysis considering large displacements. Insofar the methods are well established and readily available. The examples in the middle an at the bottom of Table 2 show some results which are determined using membrane finite elements with an isotropic St.-Venant-Kirchhoff-material allowing for large displacements and small strains.

4.1 The non-expandable chain

This small example shows a principle problem in structural optimization which results from too many degrees of freedom of the discretization.

Consider a weightless chain subjected to a distributed load of constant projected magnitude q, e.g. snow, Fig. 2. Further, consider a very rough finite element discretization of only two cable elements for half of the chain. The consistent nodal forces R_1, R_2 and R_3 are readily determined by the position of the nodes to be:

$$R_1 = \frac{1}{2} q x_2 \,; \quad R_2 = \frac{1}{2} q a \,; \quad R_3 = \frac{1}{2} q \left(a - x_2\right) \tag{6}$$

Considering the symmetry of the problem, the position of the nodes 1 and 2 have to be determined such that the structure is in equilibrium. The result is not unique and states two independent coordinates, e.g. y_1 and x_2. The remaining coordinate y_2 is then evaluated to be:

$$y_2 = y_1 \left(1 - \left(\frac{x_2}{a} \right)^2 \right) \tag{7}$$

If an additional constraint on the chain length is introduced still one coordinate remains undetermined. This means, that the problem cannot be uniquely discretized unless additional restrictions are applied. Otherwise a numerical algorithm would fail inevitably.

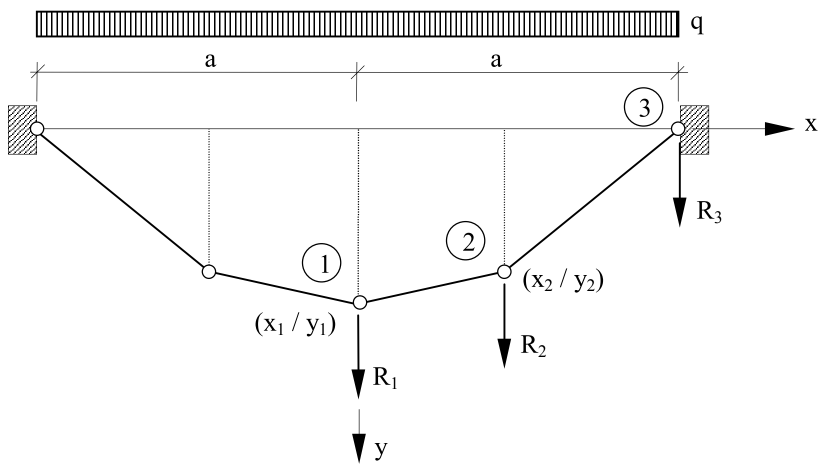


Fig. 2: The non-expandable chain

Further analysis of the problem shows that the discretization is arbitrary with respect to tangential movements along the chain. That means, that a relevant modification of the chain only can happen normal to the chain. As a consequence, each node has only one relevant degree of freedom. In the example the node 2 had two free coordinates, one too much. This result also applies to discretized surfaces of shells and membranes. Again, a node on a surface has only one relevant degree of freedom normal to the surface. In the technical practice of optimization algorithms this fact is often considered by prescribing 1D move directions which have at least a normal component to the surface or the tangent. However, this technique cannot always be applied, e.g. if the shape changes very much or if tangential movements are necessary because the whole structure expands or shrinks. In this example it would be enough to restrict the movement of node 2 to the vertical direction, fixing x_2 to e.g. *0.5 a*.

5 Pre-stressed tension structures - inverse methods II; Table 3

Like the inverted shapes of hanging models in compression, pre-stressed tension structures act in a pure state of membrane action by tension membrane forces. Again, the material is optimally used. The main difference to other form finding methods is that now the ideal state of stresses is prescribed and used as means of shape generation. The art of form finding is now to find this shape which puts the stress field of prescribed magnitude and orientation into equilibrium with respect to fixed or cable supported boundaries and external loads. The related undeformed cutting pattern is determined in a second step by compensation of the elastic deformation. In practice the amount of compensation is a question of experience or is even neglected along seams or at other reinforced regions. Compared to standard structural analysis the procedure is inverted: first the deformed structure is determined and second the undeformed. Mathematically, the problem is related to minimal surfaces [4]; experimentally, it is realized by soap films (‘soap film analogy’).

From the algorithmic point of view we are faced with the same problem we already saw at the simple cable problem above. Since the stress field is prescribed it is not related to deformation as it is usually the case in elasticity. Therefore, a deformation of a finite element discretization on the surface does not influence the stress magnitude nor direction. Again, this means that only shape modifications normal to the surface are relevant modifications. However, the reduction of the nodal degrees of freedom to a 1D direction in space is in conflict with the fact that we want to generate a free form surface in 3D space. Adjacent move directions are usually not known in advance. This method is, therefore, not very promising, although it might be used in special cases. On the other hand, this principle deficiency of the problem is the reason for the existence of all the technical procedures and algorithms which have been developed in the past. Some of them are based on dynamic relaxation [5], others, like the force density method [6] which originally was intended for cable structures, are based on special discretization and linearization techniques, as e.g. by a modified Newton-Raphson iteration [8,9].

An alternative procedure is the Updated Reference Strategy [7]. The principal idea is to add artificial in-plane stiffness to allow all three spatial degrees of freedom at any point on the surface. The effect of this regularization fades out as the procedure approaches to the final solution by an update of the reference structure.

The equilibrium condition is given by:

$$\underset{\mathbf{u}}{\delta}\,\pi = \underset{\mathbf{u}}{\delta}\,\pi_i = t\int_a \sigma : \delta\mathbf{u},_{\mathbf{x}}\, da = 0 \qquad (8)$$

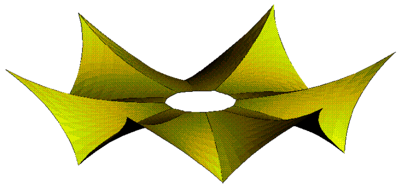
Table 3: Form Finding of Pre-stressed Membranes

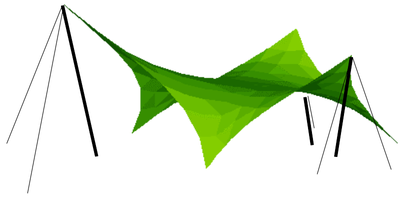
Various shapes of pre-stressed membrane structures.

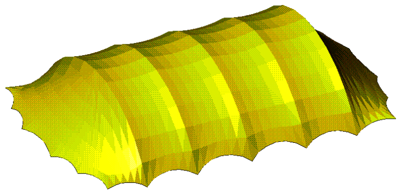
Edges are supported by cables. The resulting tension stresses due to pre-stress and elastic deformation are prescribed and define the shape.

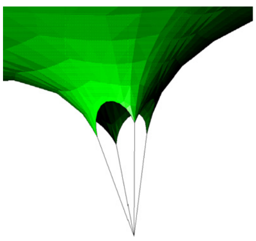
"Minimal surfaces" are defined by a homogeneous constant stress state.

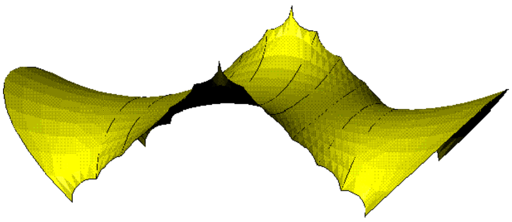
The given examples are determined by the Updated Reference Strategy. They refer to well known examples (e.g. the 'Tanzbrunnen' by F. Otto) and to student projects.

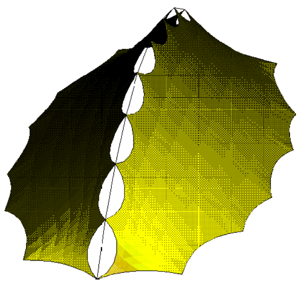


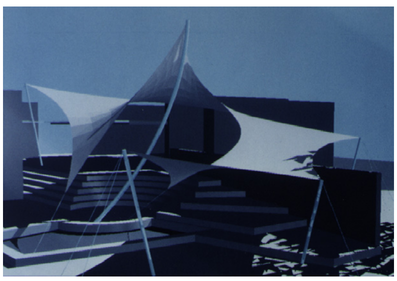


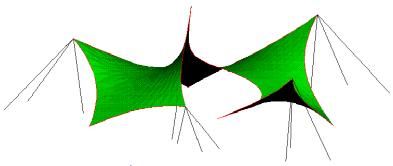


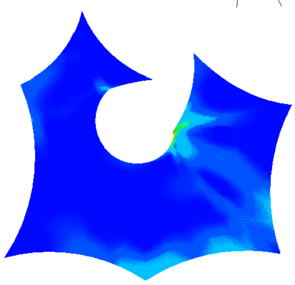


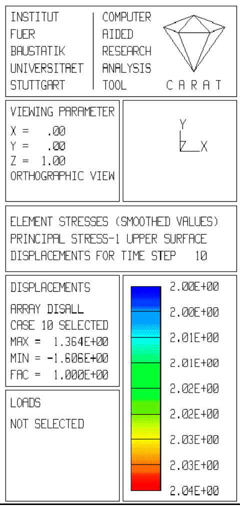












Minimal surface with constant stress distribution.
Edges are cable supported.
Above: CAD representation.

At this stage it is assumed that the structure is subjected only to pre-stress $\boldsymbol{\sigma}$. The integral is over the area a of the actual structure, where t denotes the thickness of the membrane which is assumed to be constant at any point and during the whole form finding process. The formulation can be transferred from Cauchy stresses $\boldsymbol{\sigma}$ and the gradient of virtual displacements $\delta\boldsymbol{u}$ to 2nd Piola-Kirchhoff stresses $\boldsymbol{S}$ and the deformation gradient $\boldsymbol{F}$:

$$\delta_{\mathbf{u}} \pi = t \int_A \left(\mathbf{F} \cdot \mathbf{S} \right) : \delta\mathbf{F} \, dA = 0 \tag{9}$$

Since the 2nd Piola Kirchhoff stresses $\boldsymbol{S}$ are defined with respect to the reference structure of surface area A the alternative formulation (9) has also stiffness with respect to in-plane deformations if $\boldsymbol{S}$ are prescribed instead of the Cauchy stresses $\boldsymbol{\sigma}$ which originally had to be controlled. The solution of the regularized problem (9) is used as the reference structure for the next step until the solution converges to the desired shape. Alternatively, (9) can be blended with the original problem (8) in the sense of a homotopy method to improve the speed of convergence. However, this is not necessary. The method appears to be absolutely robust and is simple and reliable in practical application.

6 Structural shape optimization; Table 4

The methods of structural optimization in particular with respect to shape optimal design are the most general optimization tools, for an overview se e.g. [10-12]. They combine highly specialized methods from different disciplines as e.g. computer aided geometrical design, computational mechanics and non-linear mathematical programming. Together they define a modular tool box which can be applied for the definition of a very general optimization problem:

$$\begin{aligned} & f(\mathbf{s},\mathbf{u}) \rightarrow \min_{\mathbf{s}} \\ & h_i(\mathbf{s},\mathbf{u}) = 0 \quad ; \quad i = 1,\ldots,\text{no. of equality constraints} \\ & \mathrm{g}_{\mathrm{k}}(\mathbf{s},\mathbf{u}) \leq 0 \quad ; \quad k = 1,\ldots,\text{no. of inequality constraints} \end{aligned} \tag{10}$$

The strain energy may be chosen as objective function f and mass as equality constraint h. The design variables may be the coordinates of a CAGD model which is used as preprocessor to generate the finite element mesh, Fig. 3. By this technique the number of design variables are reduced to very view, typically far less than hundred. Additionally, irrelevant variables can be controlled and omitted very easily.

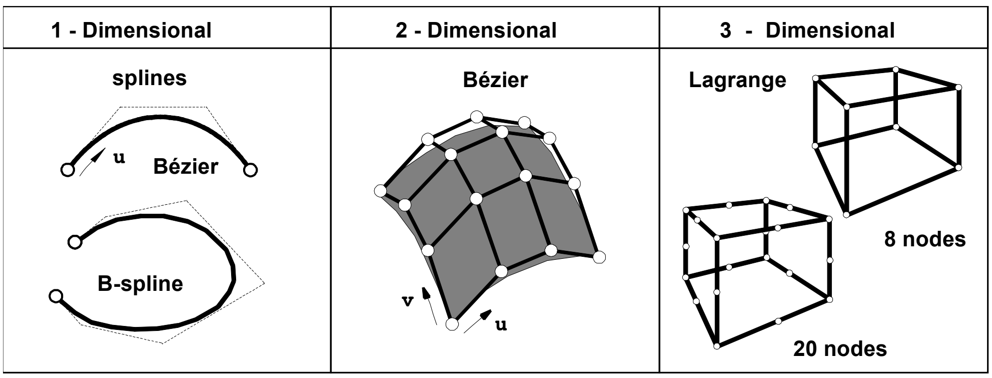
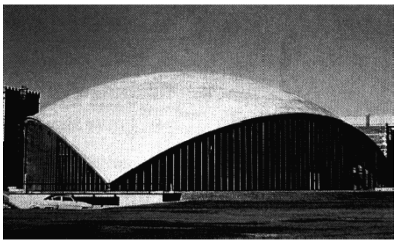


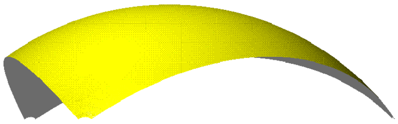
Fig. 3: CAGD model: Several design elements

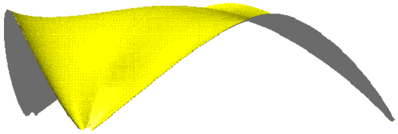
Table 4: Shape Optimization

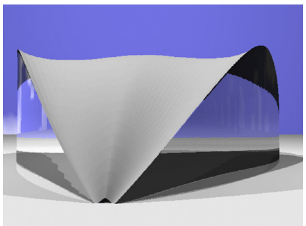
Examples of optimal shells w.r.t. minimal strain energy. The initial shapes are inspired by existing shells.



Kresge Auditorium, MIT, MA (Saarinen, 1955);
one eight of a sphere;
supported by beams at free edges

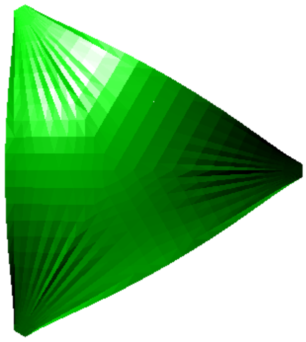




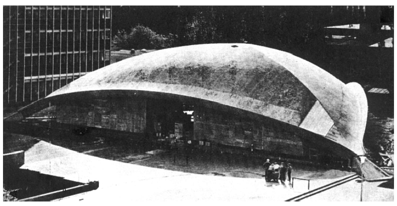


Left top: initial shape
max. deflection: 6 cm
Left middle: optimal shape by shape optimization;
variabel: vertical coordinates of design model;
free edges w/o beam;
negative curvature at edge;
max. deflection: 3 mm
Left bottom: CAD representation of optimal shape

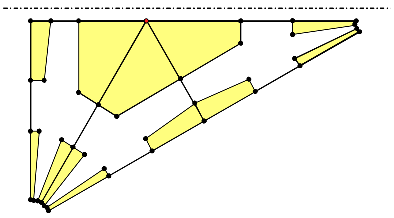
Right Top: top view of hanging model; note the wrinkles at the supports
Right bottom: side view of inverted hanging model;
also neg. curvature at edges

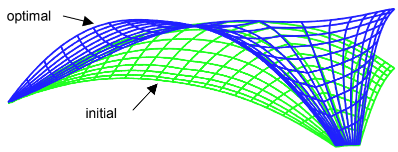


concrete shell at Uni. Of Mainz (Leonards / Mehmel, 1967)



triangular ground plan as MIT shell;
different structural concept:
forces are transmitted by 'central arches' rather than by edge beams

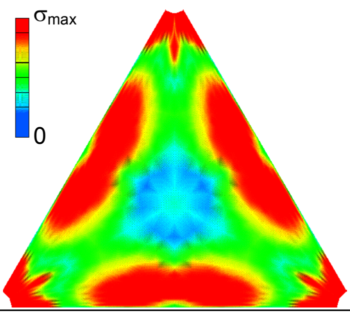


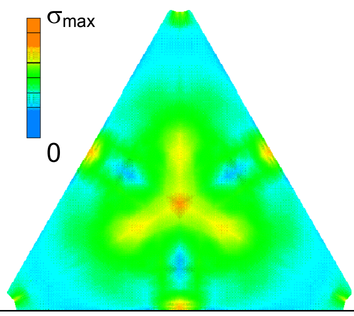


Top left: CAGD model of half the structure;
three Bézier patches

Top right: FE-mesh of initial and optimal shape; neg. curvature at edges of optimal shape;
objective: min. strain energy;
constraint: equivalent stress;
variables: vert. coordinates of CAGD model and thickness distribution;
load: dead + snow (10:1)

Bottom left/right: v. Mises equiv. stress at upper surface, initial/optimal shape, respect.





7 Conclusions

Maximation of stiffness is the common objective of the various well known computational form finding and optimization procedures, as there are: the numerical simulation of hanging models and soap film experiments, and structural optimization with its variants of topology and shape optimal design. The mechanical equivalent is 'minimization of strain energy' while restricting the total structural mass. Applications to discrete truss-like structures and continuous membrane and shell structures show the principles of optimal shapes and their variety in form as well as the general relevance of this objective for the shape design of lightweight structures.

8 References

[1] H. Isler, 'Concrete shells derived from experimental shapes', *Structural Engineering International,* **3/94**, 142-147, 1994.

[2] F. Otto and B. Rasch, *Finding Form*, Deutscher Werkbund Bayern, Edition A. Menges, 1995.

[3] E. Ramm, 'Shape finding methods of shells', Int. IASS Symposium on *Spatial Structures at the Turn of the Millenium*, Copenhagen, 1991, 59-67, 1991.

[4] S. Hildebrandt and A. Tromba, 'Mathematics and optimal form', *Scientific American Library*, 1985.

[5] M. Barnes, 'Form and stress engineering of tension structures', *Struct. Eng. Review*, **6**, 175-202, 1994.

[6] K. Linkwitz and H.-J. Schek, 'Einige Bemerkungen zur Berechnung von vorgespannten Seilnetzkonstruktionen', *Ingenieur-Archiv*, **40**, 145-158, 1971.

[7] K.-U. Bletzinger, 'Form finding and optimization of membranes and minimal surfaces', Mitteilung 3/98, Institut für Baustatik, Universität Karlsruhe, 1998.

[8] E. Haug and G. H. Powell, 'Finite element analysis of nonlinear membrane structures', Proc. 1971 IASS Pacific Symposium Part II on *Tension and space structures*, Tokio and Kyoto, 165-175, 1972.

[9] T. Suzuki and Y. Hangai, 'Shape analysis of minimal surface by the finite element method', Int. IASS Symposium on *Spatial Structures at the Turn of the Millenium*, Copenhagen, 1991, 103-110, 1991.

[10] E. Ramm, K.-U. Bletzinger, and R. Reitinger, 'Shape optimization of shell structures', a) *IASS-Bulletin*, **34**, 103-121, 1993, b) *Revue européene des éléments finis*, **2**, 377-398, 1993.

[11] K.-U. Bletzinger, K. Maute, and E. Ramm, 'Structural concepts by optimization', Proc. Int. IASS Symposium on *Conceptual Design of Structures*, Stuttgart, 1996, 169-177, 1996.

[12] K.-U. Bletzinger and K. Maute, 'Towards generalized shape and topology optimization', *Engineering Optimization,* **29**, 201-216, 1997.

[13] M. P. Bendsøe, *Optimization of Structural Topology, Shape and Material*, Springer, Berlin, 1995.

[14] G. I. N. Rozvany, M. P. Bendsøe, and U. Kirsch, 'Layout optimization of structures', *Appl. Mech. Rev.* **48**, 41-119, 1995.

[15] K. Maute, S. Schwarz, and E. Ramm, 'Adaptive topology of elastoplastic structures', *Structural Optimization* **15**, 81-91, 1998.

[16] A. G. M. Michell, 'The limits of economy of material in frame structures', *Philosophical Magazine*, Series 6, **8**, 589-597, 1904.

[17] J. Ploch, 'Entwurf und Konstruktion einer Betonbrücke unter Verwendung von Topologieoptimierungsverfahren', Diploma Thesis, Inst. für Entwurf und Konstruktion II and Inst. für Baustatik, Universität Stuttgart, 1997.