Advanced Analysis and Design of Space Dome supporting Glass Panels

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Abstract

Advanced analysis is a new design method of modeling accurately the actual behavior of a structure in an analysis so that the section capacity check is sufficient for member design. However, the application of this design method to practical structures is rare. This paper describes the design and analysis of slender domes composed of relatively slender members in which the buckling behavior of the members under compression requires a careful study and investigation. In order to meet the current design practice, the factored ultimate load is limited to the formation of the first hinge. An example of shallow dome is employed for illustration of the design method.

Introduction

A codified description on the use of the "Advanced Analysis" has been given in the AS4100[1]. This design approach has been researched extensively for the last decade. However, its application to practical structures is rare, possibly due to the complicated modeling of a structure in order to fulfil the requirements for the "Advanced Analysis". Most of these works are concentrated on the steel building frames. When using the currently available commercial programs for second-order analysis with members under high axial force, each member in the model is required to be discretised into several elements, resulting in a complication discouraging the engineers to adopt the method. Also, research on advanced analyses is mostly based on the elasto-plastic type of analysis that is too much deviated from the current practice and also difficult to verify by the current analysis method, leading to the difficulty of the practicing engineers to accept the design philosophy.

The advanced analysis allowing for second-order analysis but ignoring the plastic effects can be used for design of bare steel frames under high axial loads in members such as trusses, skeletal frames such as scaffolding, domes etc. Software capable of conducting a full ultimate analysis of rigid and semi-rigid steel building frames allowing for second-order effects and formation of a series of plastic hinges has been developed (see, Chan and Chui[2], Yau and Chan[3]) and tested well against the bench mark examples. However, this plastic design philosophy is not generally adopted by the practicing engineers who are used to the first-plastic-hinge design method in which the factored load is limited to the formation of the first plastic hinge in the structure.

This paper describes the application of the advanced analysis of a dome by the first-plastic-hinge method. The elastic post-buckling behavior of the frame is also studied in order to visualize the buckling behavior of the structure. In the complete design procedure, the effective length is not required to be assumed and the axial force-moment interactive formula needs not be applied for checking of the capacity of each member.

The Deficiency of the Most Widely used Cubic Hermite Element

The accuracy, rate of convergence and robustness of an computer program for second-order analysis depend on the non-linear solution procedure and also the accuracy of the element stiffness matrix. In this paper, the refined element satisfying equilibrium of moment and shear at mid-span of an element is used. This is termed as the pointwise equilibrium polynomial (PEP) element (Chan and Zhou[4]). By inserting an initial imperfection to the mid-span of the element using a curved element formulation, one single PEP element is capable of predicting the same load capacity as the national design code basing on the well-known Perry Robertson formula.

Previous research using the truss element with an assumed deflection curve as a half sine curve cannot be adopted directly for design since, in general, the deflection of a member can be more complicated than this sine curve assumption. The proposed element is based on a fifth order deformed shape with final deflection determined by the action of external moments, axial force and equilibrium at mid-span. It also allows for the effect of external moments in its computation of stress which cannot be avoided in practical design.

The cubic Hermite element has been most widely used in the past few decades for linear and second-order analyses. It is, however, deficient for second-order analysis when using one element per member. This can be seen very easily by a study of the simplest case of a column with both ends

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} 4\alpha + \frac{4}{30} & 2\alpha - \frac{1}{30} \\ 2\alpha - \frac{1}{30} & 4\alpha + \frac{4}{30} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

pinned, in which the element stiffness can be written as,

where $\alpha = \frac{4EI}{PL^2}$ and M_1, M_2, θ_1 and θ_2 are the nodal moments and rotations.

Solving for the condition of zero determinant for the stiffness matrix, we have α equal to 0.08 or P_{cr} equal to $12.5EI/L^2$, which is 26.6% above the Euler's buckling load. The error further increases when the buckling load is greater, as in the case of fixed ended columns.

Displacement function of the PEP Element

This paper utilises the concept of pointwise shear and moment equilibrium at mid-span for formulation of the element stiffness matrix. Thus, we have the basic displacement function as,

$$v = \sum_{i=0}^{i=5} a_i \, x^i$$

in which v is the lateral displacement, a_i is the coefficients to be determined and x is the longitudinal dimension along the element.

The following six boundary conditions are made use of in the derivation of the shape function.

$$v = 0, x = \pm \frac{L}{2}$$
$$\dot{v} = \theta_1, x = -\frac{L}{2}$$
$$\dot{v} = \theta_2, x = \frac{L}{2}$$

For compatibility,

$$EI\ddot{v} = pv + \frac{M_1 + M_2}{L}\left(\frac{1}{2} + x\right) - M_1 \text{ at } x = 0$$

$$EI\dddot{v} = \dot{P}v + \frac{M_1 + M_2}{L} \text{ at } x = 0$$

For equilibrium,

$$v = [N_1 \;\; N_2][L\theta_1 \;\; L\theta_2]^T$$

Making use of the six conditions in (3) and (4), we obtain the shape function as,
in which N_1 and N_2 are coefficients of the shape function given previously by Chan and Zhou[4]. Note that the displacement function can be expressed in a very compact and simple function of axial force and x-coordinate. Unlike the stability function and the tangent stiffness expression by Kondon and Atluri[5], the present PEP element has a consistent form for compressive, zero and tensile axial force which leads to clarity, elegance and simplicity for the element formulation.

Secant Stiffness of the PEP Element

The incremental-iterative numerical scheme based on the Newton-Raphson scheme is the most widely used non-linear solution method for structural problems. It has a fast rate of convergence and a reasonable resistance against divergence. Also, it does not suffer from the accumulative equilibrium error in tracing of the equilibrium path.

To utilize this method or its modified versions such as the arc-length method by Crisfield[6], the Constant Work method by Yang and McGuire[7] and the Minimum Residual Displacement method by Chan[8], the secant and the tangent stiffness matrices are required. These two sets of tangent and secant relations can be determined by the robust energy method. Making use of this energy principle, the total potential energy functional is first written as,

$$\Pi = \frac{EA}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \dot{u}^2\, dx + \frac{EI}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \dot{v}^2\, dx + \frac{P}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \dot{v}^2\, dx - \sum_{i=1}^{i=3} F_i u_i$$

The secant stiffness matrix can then be evaluated as the vanishing condition of the first variation of the total potential energy functional as,

$$\delta\Pi = \frac{\partial \Pi}{\partial u_i} + \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial u_i} = 0\,; \quad i = 1, 2, 3$$

The explicit form for the secant stiffness relations can then be expressed as,

$$M_1 = \frac{EI}{L}\left[c_1(\theta_1 + \theta_2) + c_2(\theta_1 - \theta_2)\right]$$

$$M_2 = \frac{EI}{L}\left[c_1(\theta_1 + \theta_2) - c_2(\theta_1 - \theta_2)\right]$$

$$P = EA\left[\frac{e}{L} + b_1(\theta_1 + \theta_2)^2 + b_2(\theta_1 - \theta_2)^2\right]$$

$$\delta^2 \Pi = \frac{\partial^2 \Pi}{\partial u_1 \partial u_j}\delta u_i \delta u_j = k_{i,j}\delta u_i \delta u_j = \left[\frac{\partial S_i}{\partial u_j} + \frac{\partial S_i}{\partial q}\frac{\partial q}{\partial u_j}\right]\delta u_i \delta u_j$$

The tangent stiffness can be determined as the second variation of the energy functional as,

The explicit form for the tangent stiffness matrix and the detailed expressions of the coefficients are available in Chan and Zhou[9] and will not be repeated here. However, it can be seen that the procedure for derivation of the two matrices required by the Newton-Raphson method is simple and robust, but results in a considerably more reliable and efficient element for second-order design and analysis of skeletal frames.

Design Criterion

In the current design practice, the maximum design load is taken as the load causing the formation of the first-plastic hinge with consideration of the interactive effect from axial force. Thus,

$$\frac{P}{\sigma_{ys} A} + \frac{EI\,\ddot{v}_x}{\sigma_{ys} Z} = 1 \text{ at } x = x_{max}$$

we have the maximum design load capacity by the following.

in which P is the axial force, σ_{ys} is the material design strength generally taken as yield stress or a certain fraction of the ultimate stress, Z is the plastic modulus, EI is the flexural constant and x_{max}

$$\frac{\partial M}{\partial x} = E\, I\, \dddot{v}_{x=x_{max}} = 0$$

is the location for maximum internal bending moment for an element determined as,

Numerical Example

In many cases, the determination of effective length of a member is not easy and a second-order analysis is needed. Further, the checking of the effect of snap-through instability over the member capacity and of the increase in member stress as a result of snap-through instability is complicated when using the linear analysis method used in conjunction with the design code.

The dome shown in Figure 1 is made of 50x50x2.5 square hollow section (SHS) as radial members and 25x25x2.5 SHS as the circumferential members. In the computer model, all the supporting circumferential nodes are pinned to rigid support but free to rotate in all directions. When under an upward wind pressure, the members of the dome are under tension that stability is not a concern and the design in this case is straight forward. For the case of downward pressure due to self-weight and when the wind blows under the dome, member and snap-through instability are required to be checked. The nodes on the outer boundary are prevented from excessive movement by the members in outer-most circumference. This arrangement will activate the arch action of the radial members in taking the pressure load. Glass panels rest on the radial members only in order to make the full use of the arch action in the members as the circumferential members can only resist loads by bending action and it is uneconomical to use them for taking the pressure loads. These circumferential members are used as bracing members against buckling in the horizontal direction and as end restraints to the radial members. The exercise here is to check the maximum design downward pressure that the structure can withstand. When using the conventional linear analysis method, the local and global axial force and moment will be used for checking of member capacity via the member interactive formulae. In the global member buckling check, the assumption of effective length for the member is difficult and may contain a considerable error. Further, the checking of the structure against snap-through instability cannot be carried out by the linear analysis method and separated checking will be tedious and inaccurate when compared with the second-order analysis method used here.

From the computer output summarized in Figure 1, the downward pressure causing the maximum stress to be equal to the material stress of 275 MPa is 5.8 kPa and the snap-through buckling load is 8.9 kPa. It can be seen that the whole design and analysis process can be completed upon the computation of stress in the analysis computer program. No assumption of effective length is required and the complicated checking of snap-through instability can be skipped. This leads to a substantial saving in time and improvement in accuracy. The original and buckling mode shape for the structure are also plotted in Figure 2 from which the weakest part of the structure against buckling is found to be at the third leveled transoms. This is expected as each radial member can be viewed as an arch subjected to vertical load and the buckling occurs near the mid-span of the member.

Conclusions

A computer-based advanced analysis method is proposed and applied to design a common type of a dome structure against member and global snap-through instability. In determining the capacity of the structure, separated individual member capacity check is not required and the effects of snap-through and member instability on the magnitification of member stress can be included in the integrated analysis and design procedure. The proposed method is expected to be widely used in the industry for design and analysis of skeletal structures with various degrees of non-linearity and modes of buckling.

Acknowledgement

The first author gratefully acknowledges that the work described in this paper was substantially supported by a grant from the Research Grant Council of the Hong Kong Special Administration Region for the project “Stability and non-linear analysis of glazed frames (Project No. B-Q080/95)”.

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