# A PARAMETRIC ANALYSIS OF THE GEOMETRY OF RETRACTABLE RECIPROCAL FRAME STRUCTURES

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## ABSTRACT

This paper reports on a parametric analysis of the geometry of a conceptual proposal for retractable structures constructed using the Reciprocal Frame (RF), a form of 3-dimensional beam grillage system. The appraisal was carried out as part of a feasibility and optimisation study. The RF has so far been used in buildings only as a static configuration, although the structure may be made to open and close by making the main beam elements (which mutually support each other) to slide one upon another. One way in which this may be achieved is by simultaneously rotating the RF beams about vertical and horizontal axes at their perimeter supports (which are held in position) whilst sliding the inner beam supports along the adjacent supporting beams. A retractable roof structure, similar to the diaphragm in a camera lens, is generated and can be used to vary the size of the central opening to match the prevailing weather conditions (i.e. fully closed, partial cover - for instance the spectators only, or completely open). Alternative ways of changing the grillage geometry to achieve retraction are also described in the paper. The interaction of the main geometrical parameters (such as the number of beams in the grid, their length, their angle of rotation in the horizontal and vertical planes and the size of the central opening in the roof diaphragm) is considered for simple polygonal grids. These relationships define the changes in geometry experienced during retraction of an RF structure and allow the load distribution characteristics of the RF grid, which vary depending on the instantaneous position of the moving beams, to be determined. From this analysis, recommendations are made for the optimum beam or truss configuration. Finally, potential problems resulting from geometrical changes during the retraction of RF roof structures are outlined with particular reference to the secondary structure between the main beams.

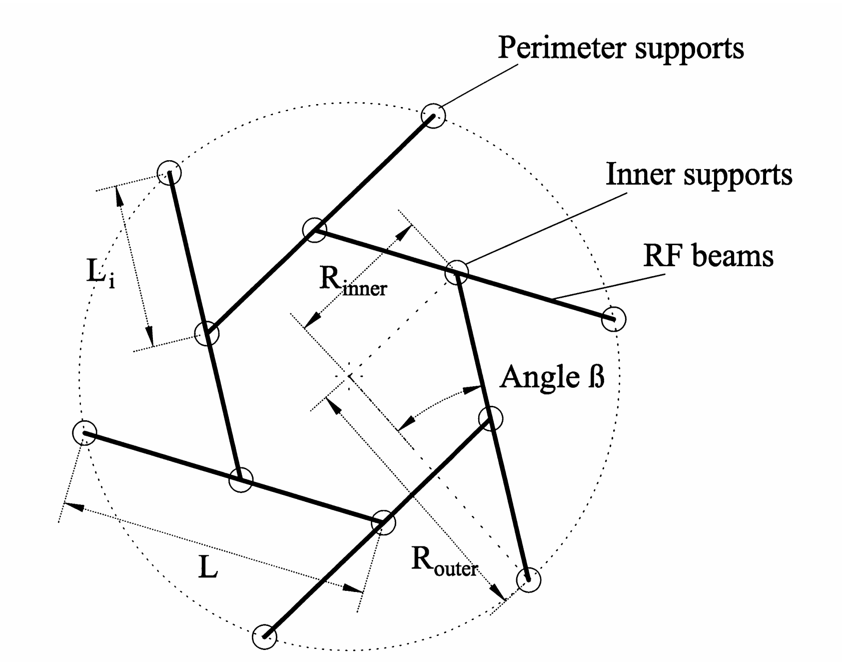
## INTRODUCTION

Reciprocal Frame (RF) is the name given to a distinctive type of architectural structure by one of its many “inventors”. This name describes rather well the nature of the basic three-dimensional beam grillage in which the beams (all shorter than the span of the structure) mutually support each other.

The purpose of the study[1] reported here was to explore the geometry of both static and retractable RFs. Small adjustments of one geometric parameter can have a considerable effect on the structure's overall shape, therefore, a sensible assessment of their interaction will facilitate design. By fully understanding the geometry and the relationships between the parameters that define it, one should be in a better position to design an RF with the properties most advantageous for long spans, and/or a practical form of retractable RF. This may be illustrated by considering beam length. As the aperture at the centre of a retractable RF increases, so the required beam span varies. The magnitude of this variation relates directly to the number of beams in the grid, thus, it may be advantageous to select the number of beams which causes the least variation during retraction.

## PARAMETRIC STUDY

Several interdependent variables (see figure 1) were considered. These included the number of beams or members (n), the angle of members to the radius line (β), the central aperture radius ($R_{inner}$), the radius through perimeter supports ($R_{outer}$), the vertical spacing between beams at the inner supports ($h_1$), the beam span (L), the distance ($L_i$) to the support for the upper beam when measured along the lower (supporting) beam and the




perimeter shape.

Figure 1. Plan of typical RF grid.

The ranges used for each of these variables were based on practical limitations: - the number of members was varied from the minimum of 3 up to 25, the angle of the beam was varied from zero degrees to the angle given by a fully opened frame. Radius to the outer supports was fixed at one unit throughout. Aperture radius was varied from zero (fully closed) to one unit (radius of the outer supports). Vertical spacing between beams at the inner supports was varied from zero (a planar frame) to 0.1 units. The perimeter of the frame can be any regular or irregular shape, however, in general, stadia are either oval or circular in shape, and to simplify the geometry a regular circle was used throughout this study. Because the variables are interdependent, once any three of the above variables have been set the others may be calculated using equations derived by Chilton and Choo[2] and Dean[3].

The parametric studies were executed in three stages. Initially, the Reciprocal Frame was assumed to be planar, i.e. where the vertical distance between beam centre lines at the inner support was zero. The second stage was to analyse the RF frame as a three-dimensional grillage, i.e. to give height to the structure by spacing the beams vertically at the inner supports. Because most of the results obtained from the planar analysis could be transferred directly to the three-dimensional model, here, the main objective was to investigate how the height of the structure alters with a varying aperture. The last stage examined how the structure distributes loading to the supports and the forces and moments involved in this load transfer.

When plotting the relationships graphically, in order to illustrate all three variables, two are represented on the axes, and the third as a series of lines, one for each value. It should be noted that for graphs involving angle β, the maximum angle for frames with only 3 beams is 30 degrees whilst for frames with 6 beams, or more, it is 60 degrees. This is because the angle of the beams when fully opened is dependent on the angle between supports. Where n = 3, because the supports are so far apart, a fully opened frame gives a beam angle of only 30 degrees, where n = 4 this is 45 degrees and where n = 5 the maximum angle is 54 degrees.

## PLANAR FRAME

It can be seen from figure 2 that as the frame opens (i.e. as the angle β increases) the beam span changes. Three distinct types of behaviour can be identified, depending on the number of beams present in the RF grid.

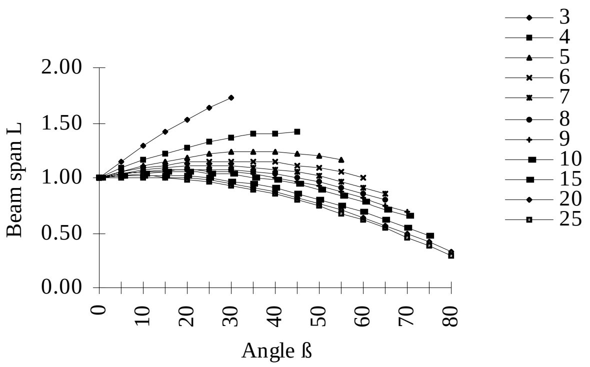



Figure 2. Variation of beam span L for different beam rotation angles β and number of members.

The first type, frames with a number of beams less than 5, have a minimum beam span when the frame is closed and this increases to a maximum when the frame is fully open. This is due to the spacing between perimeter supports being greater than the radius of the structure. Those with more than 10 members perform in the opposite way, as the beam spans are longer when the frame is closed and (apart from a very small initial lengthening) get shorter as the frame opens. Frames having from 5 to 10 members have a beam length that initially increases as the frame opens, reaches a maximum at some intermediate angle, and then decreases again as the frame opens to its full extent. This variation in beam length has important implications for retractable Reciprocal Frames, both from an architectural point of view and also structurally. In all cases, the beam length has to be longer than the actual beam span at some stage in the retraction process.

Additional beam length must be accommodated at the perimeter supports (by allowing the ends of the beams to slide outwards over the supports during retraction) or by using a telescopic beam. An alternative for three-dimensional grids (but not possible for flat grids) is to accommodate the additional length at the inner supports by allowing the upper beams to slide over the supporting beams. All of these are possible solutions but they tend to detract from the simplicity of the basic RF structure and it is, therefore, advantageous to keep the beam length variation to a minimum. Frames with between 5 and 7 beams have the smallest variations, of approximately 10 to 15%, and it would seem prudent to limit the number of beams to this range when designing a retractable Reciprocal Frame.

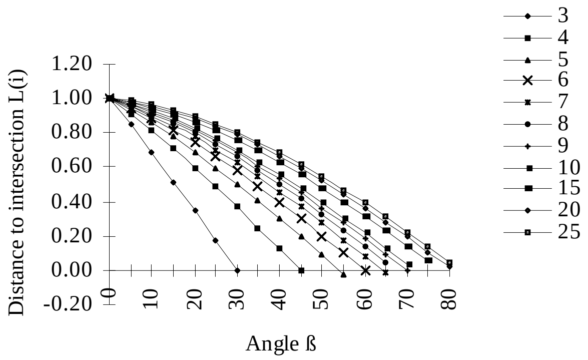



Figure 3. Distance $L_i$ to the inner support along the supporting beam for different beam rotation angles β and number of members.

From figure 3, one can see that, in theory, when the retractable RFs are fully closed, the beams all meet at the centre of the roof (i.e. $L_i = 1$). The distance from perimeter supports to the inner supports is equal to the radius of the structure, although this cannot necessarily be achieved in practice. As the central aperture increases, the inner support of each beam moves along the adjacent supporting beam towards the perimeter support. The rate at which this motion occurs is not constant and this effect is especially pronounced for the grillages with many members. At small apertures, the movement is slow compared to the angle through which the beam rotates. As the frame opens to its full extent the movement along the supporting beam is greater, for the same change in rotation angle. The variation in the rate at which the beam inner support moves along the supporting beam will have implications for the drive mechanism controlling a retractable frame, which might, for example, use a rack and pinion system running along the length of the beam. Greater torque will be required when the frame is first opening than at the larger apertures, and hence some kind of variable gearing system may be required.

In figure 4, the ratio of distance to the inner support position compared with the beam span is shown for different opening angles β and various n. This relationship influences the magnitude of load distribution that takes place round the RF. If the inner supports of the beams are close to the perimeter supports, then a greater percentage of the applied load will be transferred to the perimeter support as opposed to the inner support of the next beam.

Due to the reciprocal nature of the structure, if the beam inner supports occur close to the inner ends of the supporting beams, then the majority of the load is transferred around the structure, perhaps many times, before it is taken to the perimeter supports. This generates high shear loads at the inner supports and these are sometimes greater than the sum of all the imposed loads.

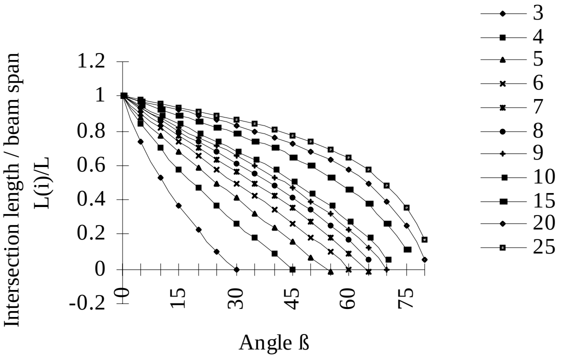



Figure 4. Ratio of distance $L_i$ to the inner support along the supporting beam to beam span length L for different beam rotation angles β and number of members.

Again there are different trends depending on the number of beams. For the frames with few members, the beam span increases as the frame opens and the relative position of the inner support at first moves quickly towards the perimeter. However, as the aperture increases, the rate of movement decreases slightly. In contrast, for frames with many members the beam span decreases as the frame opens and the inner supports are initially slow to move away from the inner end of the supporting beam. It is really only at the fuller apertures that the inner supports shift rapidly towards the beam perimeter supports. For example, when n = 3, the inner support is halfway along the span when the aperture is 40% of its maximum but it is not until the aperture is 85% that the inner support reaches the midpoint of the supporting beam when n = 15.

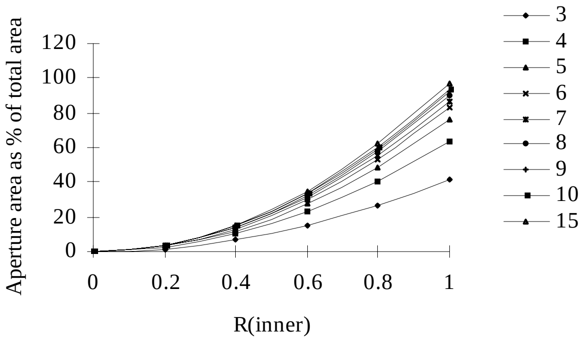



Figure 5. Aperture area for different $R_{(inner)}$ and number of members, n = 3 to 15.

The study found that the relationship between the rotation angle β of the beams and the inner (aperture) radius of the structure is almost directly proportional for the frames with a low number of beams but is slightly less so at large opening angles when there are more beams. Therefore, for the purpose of this parametric study, trends relating to the beam angle β can be applied approximately to the aperture, a large angle reflecting a large aperture. As shown in figure 5, the aperture area, for any given inner radius, varies according to the number of beams making up the structure. However, the aperture is only an approximation of a circle - it is a regular polygon, with number of sides equalling the number of beams. Therefore, for a small number of beams, the aperture formed has an area much smaller than a circle or a polygon with more sides (e.g. for n = 3 the aperture when fully open is an equilateral triangle of only 41% the area of the encompassing circle).

## THREE DIMENSIONAL RECIPROCAL FRAME

Figure 6 shows the relationship between the height (H) at the inner support of the beam (i.e. above the level of its perimeter support) and the relative position of that support $L_i/L$, for various vertical spacing between beams, in an RF where there are 6 beams. This reveals that for small apertures, (i.e. with $L_i/L$ close to 1) the height of the structure is far greater than the vertical spacing between individual members. Starting with a fully opened structure, as the roof is closed it rises only slowly initially.

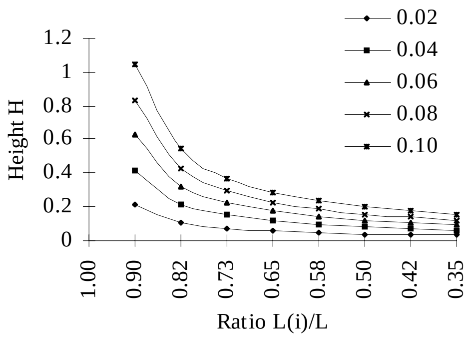



Figure 6. Relationship between the height (H) at the inner support of the beam (i.e. above the level of its perimeter support) and the relative position of that support $L_i/L$ along the beam - six beams with different vertical spacing between their centre lines.

However, significant changes in roof height may occur as the central aperture becomes small and this could be a problem for the designers of retractable RFs. Members would be necessarily longer than in a flat RF system. The height at the inner supports (and hence the rise of the structure) is in direct proportion to the number of members and to the vertical spacing ($h_1$) between beam centre lines where they meet at the inner support[4]. For example, if the number of members is doubled, then for any given aperture, the overall height of the structure is also doubled. Likewise, if the vertical distance between beam centre lines is doubled then again the overall height of the structure is also doubled. In previously constructed static RFs where the beams rest upon each other, the upper beams are often notched at the inner support. The vertical spacing between beam centre lines is thereby reduced and, consequently, the overall height of the structure. This modification may also be applied to retractable RFs. The most efficient shape for the RF beams or trusses in bending is a “fish-belly” longitudinal section derived from the influence line for the applied point load from the adjacent beam. At the inner end of the supported beam, the bending moment is zero, therefore, the vertical distance between beam centre lines can be reduced by minimising the structural depth of the supported beam, thus limiting the rise at the centre of the roof when closed.

## LOAD DISTRIBUTION FOR A PLANAR RECIPROCAL FRAME

The members all support one another. Applied loads are transferred around the structure a number of times (reducing in magnitude each time as the outer supports take a proportion of the load). This is very different from most structures. Formulae describing the load distribution characteristics of the RF have been derived by Chilton and Choo[2], and Dean[3], and these were used to obtain the beam reactions in figures 7 and 8. These show how a point load of one unit, acting at the inner support of one beam, is distributed around a frame of respectively 6 or 20 members.

When the aperture of the RF is small, the majority of the applied load passes to the adjacent beam and from beam to beam, therefore the inner support reactions are high for all beams. The perimeter reactions are reasonably similar. However, for an open RF the beam inner supports are close to the perimeter supports and the applied load transfers more directly to the perimeter. Therefore, the inner reactions are much smaller and both they and the perimeter reactions are less evenly distributed. In some cases the inner reactions may be greater than the applied load. For instance, in the frame with only 6 beams, when the rotation angle is 5 degrees ($R_{inner}$ = 0.1), the reactions at the inner supports vary between 1.8 to 1.2 units, when the total applied load is only 1 unit.

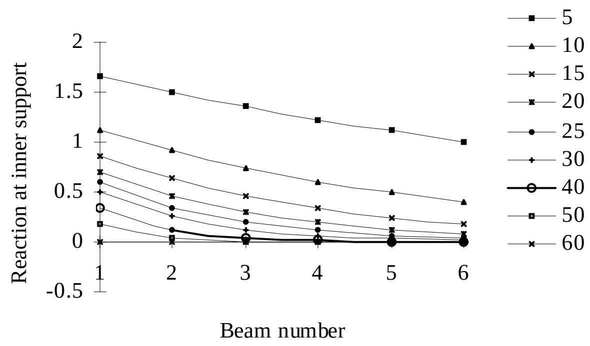



Figure 7. Beam reactions at inner supports for a unit point load acting at the inner support of one of six beams, for beam rotation angles β = 5 to 60 degrees.

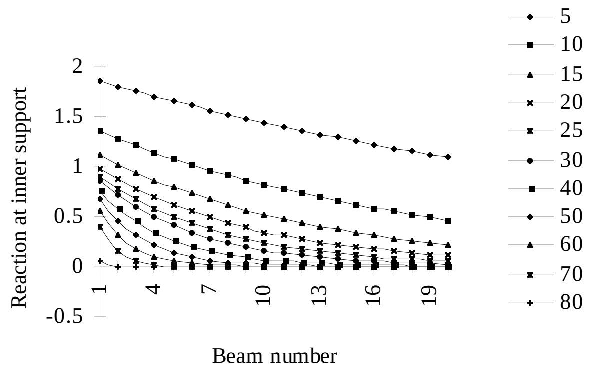



Figure 8. Beam reactions at inner supports for a unit point load acting at the inner support of one of 20 beams, for beam rotation angles β = 5 to 80 degrees.

Where there are 20 beams it can be seen from figure 8 that the reduction in load being transferred after one complete revolution is approximately the same as it is with six beams. This might at first seem improbable but, when forming an aperture of the same radius with more beams, the inner supports are further from the perimeter (i.e. $L_i / L$ is greater). Therefore, the reaction at the inner support is greater for each beam. The main difference between this and the previous example is that the reactions at each perimeter support are substantially less.

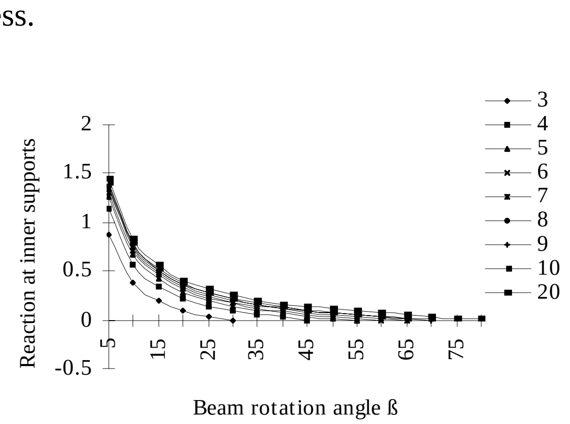



Figure 9. Unit load distributed among all beams and applied at the inner supports.

With point loads acting on each inner support all inner support reactions are equal. Figure 9 shows the beam inner reactions (with a total load of 1 unit on the whole frame), for various aperture openings and number of beams. It can be seen that for any given angle of rotation the reaction at the inner supports is greater when there are more beams and that for rotation angles less than 15 degrees these reactions are high. Again this suggests that retractable RFs should be designed with as few members as possible but also that the central aperture should not to be small.

## CONSTRUCTION

The construction of an RF roof, whether static or retractable must commence with the erection of the main structural members i.e. the skeletal frame of the RF. At least one temporary support is required for the RF beams or trusses until all are in place and supporting one another. For large RFs the size of the main trusses both in terms of their dimensions and weight could be at the limits of cranes available today. Instead sectional construction might be required, each truss possibly being split into two or three parts. However, it would be possible to fabricate the members of a medium span RF at ground level and lift them into position with a crane. For larger span RFs a network of long span secondary roof trusses would be required to which roof decking or glazing would then be attached. Although it might be more economical to assemble the individual sectors of the RF roof from a proprietary space grid system.

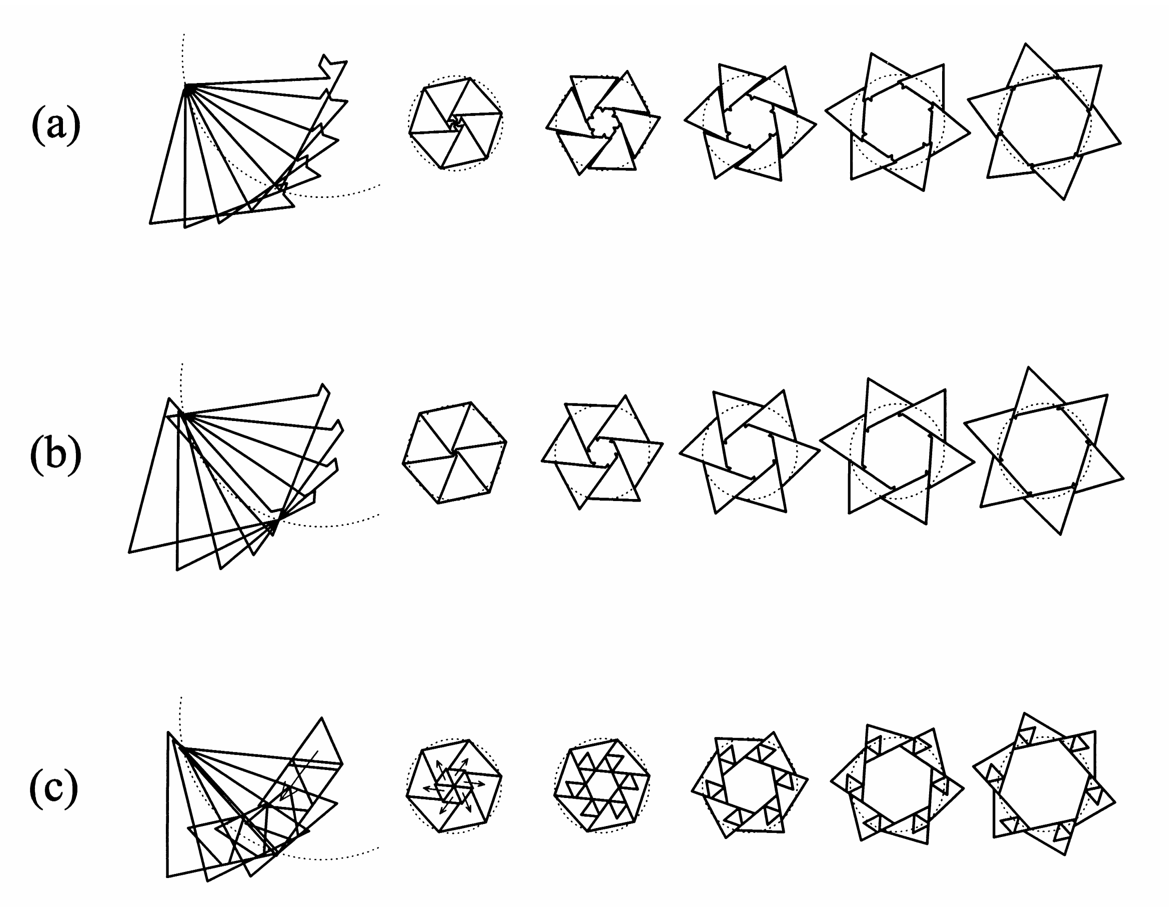


Figure 10. Alternative retraction geometries for an RF roof; (a) rotation about the perimeter supports, sliding at inner supports; (b) sliding and rotation at the perimeter supports whilst inner support moves along the line of the supporting beam; (c) hybrid system as (b) but with radially sliding segments to cover the central aperture of the RF.

Figure 10 shows three alternative proposals for retractable RF roofs. In the first, figure 10(a) the roof sectors rotate about the perimeter supports (that are fixed in position) and are free to slide over the adjacent sector at the inner supports. However, in this case, gaps open up between the roof sectors near the perimeter during retraction and this might be unacceptable if it rained when the roof was partially opened to expose only the field of play for a football match, for example. In the second case, figure 10(b), the inner support of each roof sector moves along the edge of the adjacent supporting sector. It does not slide over the adjacent sector but both sliding and rotation occur at the perimeter support. There is no overlap of adjacent sectors and the only opening occurs at the centre of the roof where it is wanted. In both of these solutions a small central aperture has been covered by rigid cantilever sections of roof attached to the main RF sectors. However, from the parametric study small apertures were found to be undesirable. Also a considerable area of ground would be required outside the stadium to “park” the retracted roof sectors. Therefore, a third option, figure 10(c) proposes a hybrid system in which a larger central aperture is covered by segments that retract back over the main RF retractable roof. This allows partial opening of the roof without moving the RF, improves the load distribution characteristics of the RF (as the inner supports are nearer the perimeter supports) and also reduces the intrusion of the retracted roof sectors into the areas surrounding the stadium.

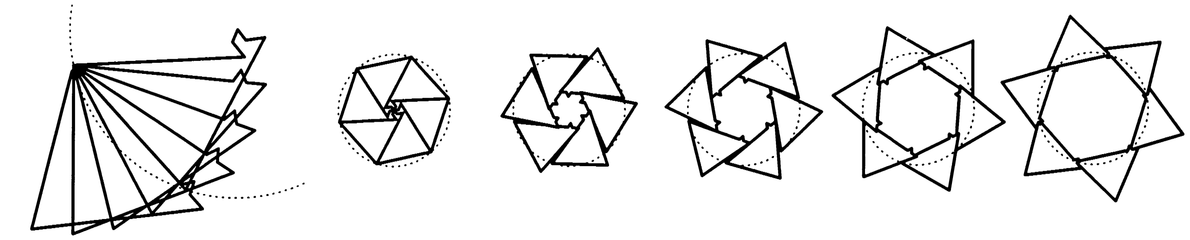
## SUPPORTS

To produce an RF with a minimum change in beam span during retraction, the ideal number of beams is six. Depending on the precise configuration of primary and secondary members it is possible, therefore, that practically the entire weight of the roof and applied loads, is taken by just six main supports at the roots of the members. This would be advantageous, as large and complicated bearing systems would be required and it would, therefore, be better to keep their number to a minimum.

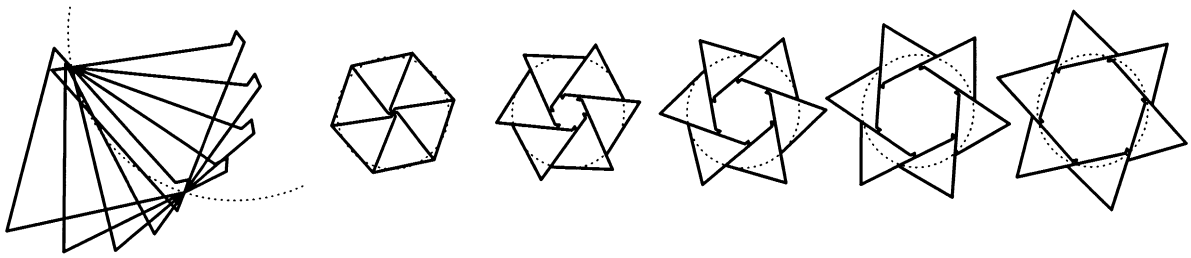
## AESTHETICS

The Reciprocal Frame is undoubtedly a visually dynamic form, and its capability to retract, as an inherent feature of its form, results in an elegant sliding motion. When used as a roofing system, attention is drawn to the centre, instilling a dynamic feel to the structure, and the central aperture allows for roof lights etc. to be installed. The asymmetric layout is visually stimulating and the architectural use has much potential. In many existing retractable roofs the retractability has been forcibly engineered into the structural from. However, retractability and the RF go hand in hand and the dynamic form of the RF leads directly to thoughts of movement. It is this natural link that would make the RF such an attractive choice for a retractable roof. The movement would be elegant at all stages and would enhance as opposed to obstruct the beauty of the structure. It is this aesthetic appeal that potentially gives the RF an edge over other retractable forms.

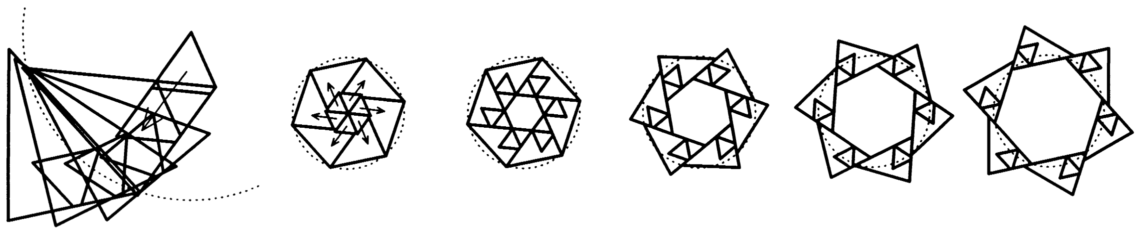
(a)



(b)



(c)



## SAFETY

Working as engineers, safety should be at the forefront of our thoughts. This is especially so for stadia, potentially towering above 100,000 people. The main structural disadvantage with isolated Reciprocal Frames is that if any one principal beam fails, then progressive, catastrophic failure of the entire structure proceeds. This might be too great a risk, particularly in the current political climate of frequent terrorist attacks. Some suggestions have been made about ways to prevent this[2]. One solution might be to design the beam / trusses to act as propped cantilevers under normal loading conditions but to act entirely as cantilevers from the perimeter support under accidental loading. Another possibility is, as suggested above for other reasons, to design the roof as set of space grid panels with multiple supports between them

## CONCLUSIONS

It has been shown that Reciprocal Frames could offer an attractive solution to the problem of designing retractable roofing systems. From the parametric study of the geometry it was concluded that, to minimise the variation in distance between the primary RF beam supports during retraction the number of beams should be 5,6 or 7. This number of beams also gives an approximately constant speed of movement of the beam inner supports during retraction. To reduce the problems produced by the changing three-dimensional geometry during retraction the RF roof should be as near planar as possible. Analysis of the load distribution characteristics also found that more beams did not necessarily lead to a more economical structure and also suggested that the central aperture should be as large as possible.

In terms of efficiency, the RF is not ideal for long spans. The use of RFs will probably be restricted to cases where economy of materials is not of the highest priority and where the ability to retract decides which roof system is used. Although, in theory an RF could be used for large structures including stadia, it is envisaged the first retractable Reciprocal Frames will be used for medium span buildings such as swimming pools.

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