Effect of Joint Stiffness on the Buckling Behavior of Reticulated Dome Structures

A. Dorfmann

Institute of Structural Engineering
University of Applied Sciences
Peter Jordan Street 82
1190 Vienna, Austria
Tel: +43-1-47654-5255
Fax: +43-1-47654-5292
e-mail: dorfmann@ikiserver.boku.ac.at

ABSTRACT

Joint connections in reticulated space structures can be categorized into four groups: member end bolting, shear bolting, connection through groove or key joints and on site welding. The “Triodetic” joint, an example of groove connector, is a very cost effective structural solution and is used frequently to allow for short production and erection time. No bolts or welds are required to connect round structural tube elements to a central hub. The second type of joint system referenced in this study is a double plate hub system. Extruded aluminum “I” beams are rigidly interconnected with top and bottom gussets using high strength stainless steel bolts. These two connector systems are both characterized by strong out of plane bending resistance. The structural difference between the systems lies in their in-plane response. In this paper accurate numerical models are formulated capable of evaluating the influence of the in-plane stiffness of joint connections on the global buckling behavior of reticulated dome structures. It is shown that the determination of the bifurcation point along the primary equilibrium paths is strongly influenced by the amount of rotational stiffness provided by the connectors.

INTRODUCTION

A reticulated or latticed dome structure is defined by Buchart [2,3] as a form resulting from approximating a solid shell surface by a framework of relatively short linear structural members. The State-of-the-Art report on latticed structures published in 1976 by the ASCE Journal of the Structural Division, defines a lattice structure as a structural system in the form of a network of elements (as opposed to a continuous surface). Rolled, extruded or fabricated sections comprise the member elements capable of transferring in-plane forces from joint to joint. The overall stiffness is similar to that of a thin shell where loads are carried primarily by membrane action. Reticulated dome structures using the structural systems described above have proven to be an efficient solution for covering multipurpose sport facilities and other large assembly buildings.

The buckling behavior of triangulated space structures has been the subject of numerous studies over the last four decades. During the 1960’s and 70’s the emphasis was on the development of simplified closed form solutions using continuum shell analogy, [2-4]. In these studies, simplifications were assumed among others, in the description of joints, loading and geometric imperfections.

More recently, advancement in computer technology and the development of faster numerical methods for the treatment of non-linear problems have led to the increased use of the finite element method in the buckling analysis of lattice structures, [5, 14, 18, 20]. Numerous publications have been presented in this area correlating physical test results with nonlinear finite element models, [8, 13, 19]. Experimental tests in combination with numerical results have enabled engineers to develop accurate models of these structural systems, including the effect of geometric imperfections, material non-linearity, and boundary conditions, [9, 11, 21]. Research also expanded to include post-buckling behavior, bifurcation buckling into non-symmetrical modes, and the effect of non-uniform loading, [12, 13].

Limited amount of information is available, however, on the influence of different joint systems on the buckling behavior of aluminum space structures, [10, 15, 16, 22]. Hanaor [10] provides a systematic classification of various joint systems. The summary classifies joints in terms of the three main components: the member cross-section, the shape of the hub (if present) and the connection method. Member cross-section typically have a circular or a rectangular hollow shape. They can be of hot or cold formed sections or extruded aluminum sections. The hub category includes spherical, polyhedral, cylindrical and prismatic hubs; plate hubs and molded hub connectors. The member-joint connections contain a very large number of variants. Connections can be achieved with the use of threaded components, end or shear bolts, connection through a specially molded member tip and site welding. Each of these details contribute to the joint stiffness which in most cases can only be determined by testing.

Interesting is the class of descriptive features which include items such as joint rigidity. Hanaor found that joint rigidity plays an important role in the behavior of space frames under load. Even for concentric member connections, large rotational stiffness of the connector enhances ductility and strength in reticulated structures, particularly in shallow domes. Joints which are ideally pinned may give rise to joint instability, particularly in the presence of geometric imperfections. Lopez and Orrison [15, 16] used statistical regression techniques to investigate the effect of joint stiffness on the critical buckling load. Their results show the importance of the joint stiffness in determining bifurcation loads in reticulated dome structures. The buckling load proved to be highly sensitive to changes in the joint stiffness for structures with low degree of connection fixity. For structures with fixed or semi-rigid joints, changes in the joint stiffness induce very small changes in critical buckling loads. Finally, the experimental study in [22] summarizes tensile, compressive and bending tests of circular pipe elements connected to Triodetic joints. It is shown that the stiffness ratio of out-of-plane to in-plane bending rigidity is approximately 40:1. Test results of a 10 meter diameter aluminum shallow dome show the ultimate failure mode as excessive rotational on a number of joints, a direct consequence of the low in-plane rigidity of the Triodetic joint.

The objective of this study is to evaluate the importance of in-plane rotational stiffness on the global buckling behavior of space structures. For a consistent analysis of domes with varying in-plane joint rigidity, numerical models are formulated capable to obtain a non linear static equilibrium solution for global buckling behavior.

JOINT SYSTEMS AND NUMERICAL MODEL

An efficient and optimized design of reticulated shell structures must consider the direct member forces, displacement characteristics, the influence of bending, imperfection sensitivity, both local and global buckling behavior and the particular connector used. The joint system is the most complex part and can be categorized by its three main components: the member cross-section, the shape of the hub and the connection method. Joints can be further classified into rigid, semi rigid, low rigidity and pinned connectors depending upon the degree of fixity in each of two orthogonal directions of rotation: rotation in a tangent plane and rotation in a perpendicular plane [10].
Joint systems frequently used in the design of reticulated dome structures are the Triodetic connector and the double plate hub system pioneered by Temcor [17]. The Triodetic system shown in Figure 1 was developed in Canada in 1953 and consists of a central hub with a number of profiled slots around its periphery, [7]. The hub is produced as an aluminum extrusion, it is a low cost operation and thus different shaped hub sections can be economically produced. The tubular members are manufactured by first cutting the elements to the required length and correct angles and then by a simple pressing operation to form the coined end profile. The connection between member and hub is accomplished by inserting the ends of the precut and stamped tubes into toothed slots around the periphery of the hub. In the second system referenced, aluminum beam elements are rigidly connected with top and bottom gussets to ensure high moment transfer at the joints. The structural members have a symmetrical "I" section with extruded grooves for connection of the interlocking panel and batten system [17], see Figure 1. Hanaor [10] classifies the system pioneered by Temcor as a fully rigid joint in both directions. The Triodetic joint is classified in the same survey as having a rigid connection for rotations in the perpendicular plane but as low rigidity for in-plane rotations since the bending stiffness is provided by squashed tube ends. It is the difference in their in-plane bending response which is analyzed in this study. The main assumptions made in this analysis are linear elastic material behavior and rectangular cross-section of the frame members. This replacement in cross-sectional shape is necessary to be able to use of the shelf plate/shell elements with varying thickness in the beam direction and for a consistent analysis of connectors with very low in-plane bending stiffness to fully fixed end conditions.

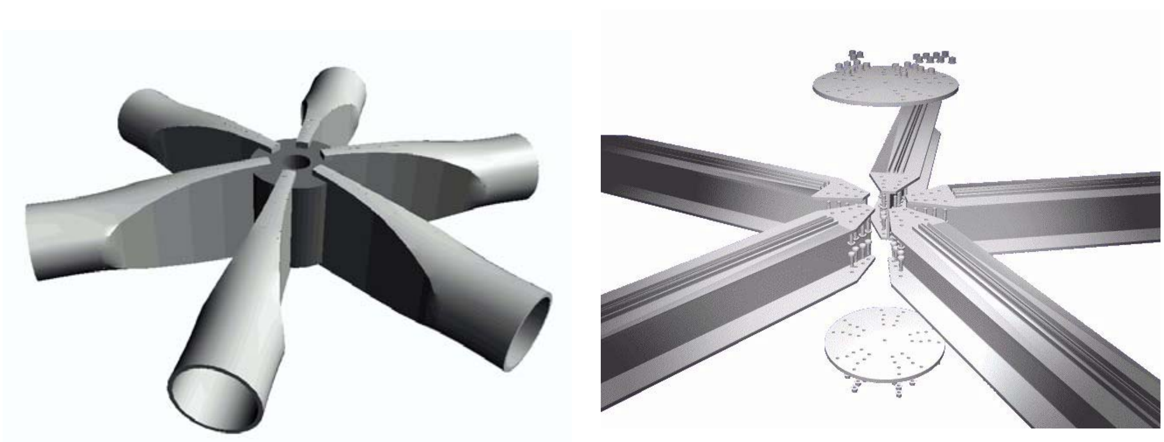

Figure 1: Connectors used in Reticulated Dome Structures.

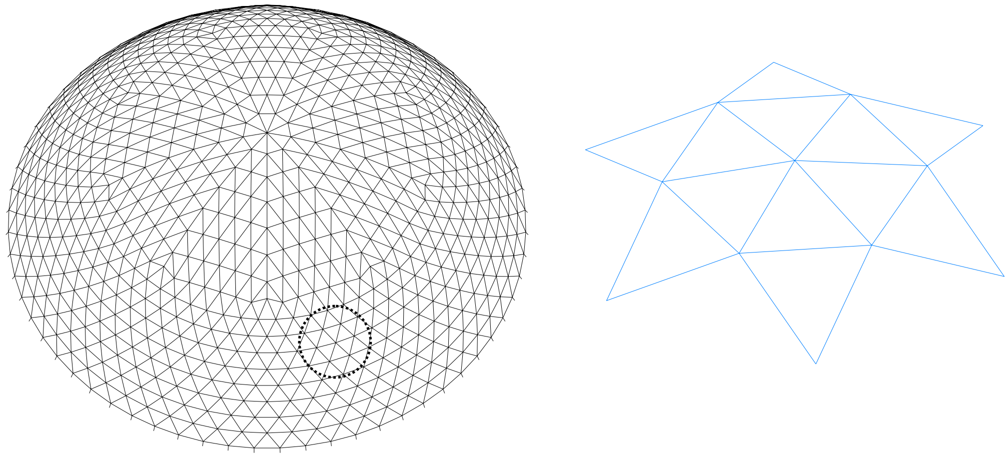

Figure 2: Reticulated Dome Structure and Selected Simplified Model.

First, an idealized configuration is selected to verify the numerical model and to justify the use of a rectangular cross-section. For that purpose, consider the aluminum dome structure shown in Figure 2 and extract a limited number of beams and joints to built a simplified but at the same time realistic joint model. Since the geometric configuration and dimensions have not be altered, the model can be considered representative of the remaining structure. The outer most joints in the model are assumed fully restrained and thus simplifications are assumed with respect to the full 3-D structure where the corresponding nodes are allowed to displace. The most important consideration in the model lies in the selection of the joint detail and the member type. Let the length of the individual members be equal to 4.2 meter and the angle between them 60°. The rectangular cross-sectional dimensions are equal to a height of 0.12 meter and a thickness of 0.025 meter, see Figure 3. In addition, it is assumed that each beam is modeled using five plate elements. The center element has a constant thickness of 0.025 meter. Attached to the center plate are two 0.075 meter long transition sections with varying thickness and finally

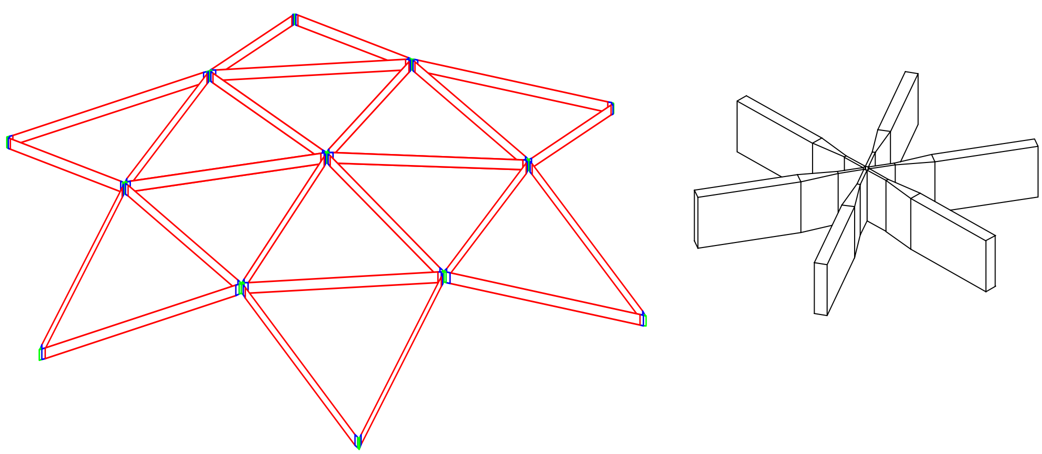

Figure 3: Finite Element Model and Joint Detail.

the two end section with constant and reduced thickness when compared to the center part, see Figure 3. The transition plate elements have a linearly varying thickness in the beam direction starting from 0.025 meter on one side to values of 0.025, 0.0125, 0.00625 and 0.00125 meters. Thus, the in-plane bending rigidity of the transition elements are reduced by a factor of 1, 8, 64 and 8000, respectively. Concentrated vertical loads are applied at each joint and the Riks method in ABAQUS [1] is used to successfully track the load displacement response of the structure. The magnitude of the applied load is scaled during the solution process by a single factor to make sure that the solution remains bounded and progresses in a consistent direction. Plots of the total applied load versus the vertical displacement of the center node are shown in Figure 4. The load corresponding to the bifurcation point along the primary load paths is reduced as a function of the in-plane rotational stiffness. Furthermore, it can be observed that the reduction in load carrying capacity depends almost linearly on the reduction in thickness, see Table 1. To illustrate the different failure modes, deformed grid plots are shown in Figure 5 for the structure with no thickness variation (Stiffness = 100%) and the structure where the in-plane stiffness is reduced by a factor of 8, (Stiffness = 12.5%). Both models shown have a vertical center displacement equal to 0.075 meter and all displacement components are amplified by a factor of 5. For the model with a stiffness reduction to 12.5% in the transition section, the applied load is about half and the failure mode is associated with excessive rotation of the weakened joints. This deformation mode can qualitatively be compared to the ultimate failure mode of the 10 meter Triodetic dome structure in [22].

Thickness Ratio	Stiffness	Buckling Load (kN)
1	100%	2184
0.5	12.50%	963
0.25	1.562%	422
0.05	0.012%	30

Table 1: Buckling Loads versus Thickness Ratio

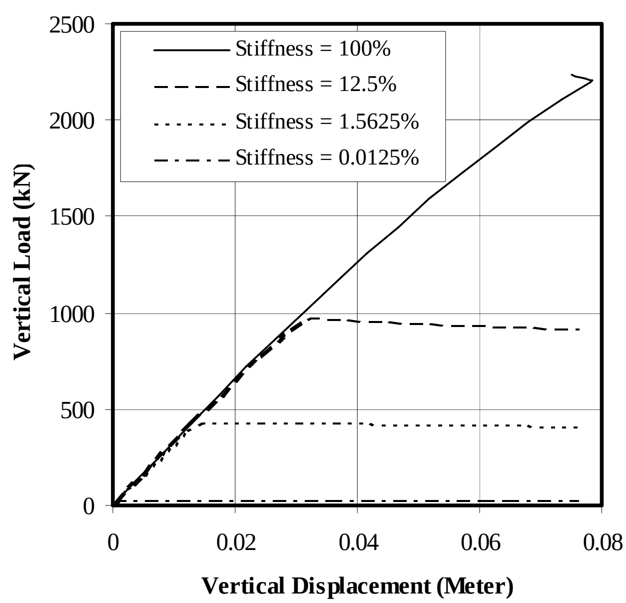

Figure 4: Buckling Loads versus Vertical Deflection at Center Node.

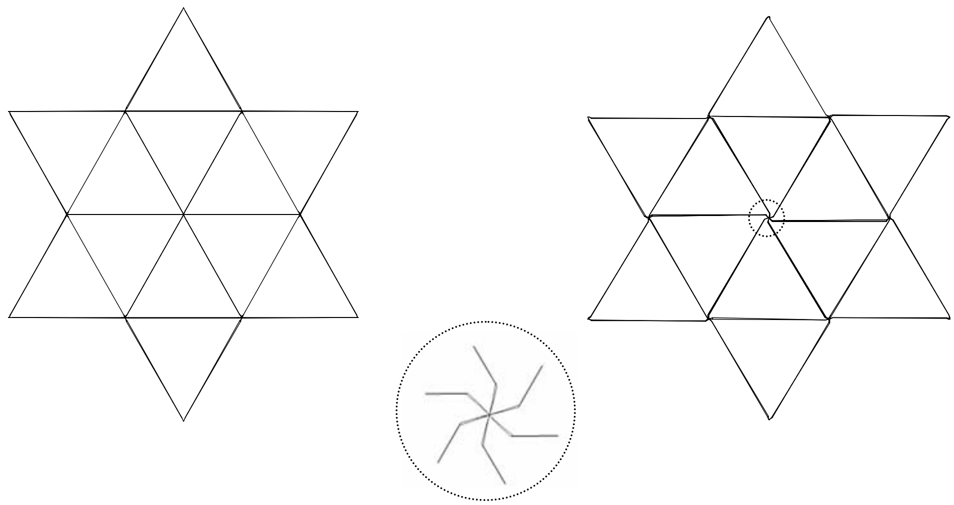

Figure 5: Failure Mode for Models with Thickness Ratio Equal to 1 and 0.5.

APPLICATION

An aluminum dome dimensioned to withstand a prescribed vertical load is used to evaluate the influence of in-plane joint connection fixity and to validate the numerical model proposed in the previous section. The design process for reticulated space structures requires first the development of an optimum geometry to satisfy the functional requirements, second accurate information regarding the magnitude and distribution of applied loads, and third, structural optimization to minimize the overall weight. The optimized design solution is shown in Figure 6 and consists of a spherical dome with a constant radius of curvature of 25 meters, a diameter of 24 meters, a total center height of 3 meters and a total of 168 members. Each beam is assumed to have a rectangular cross

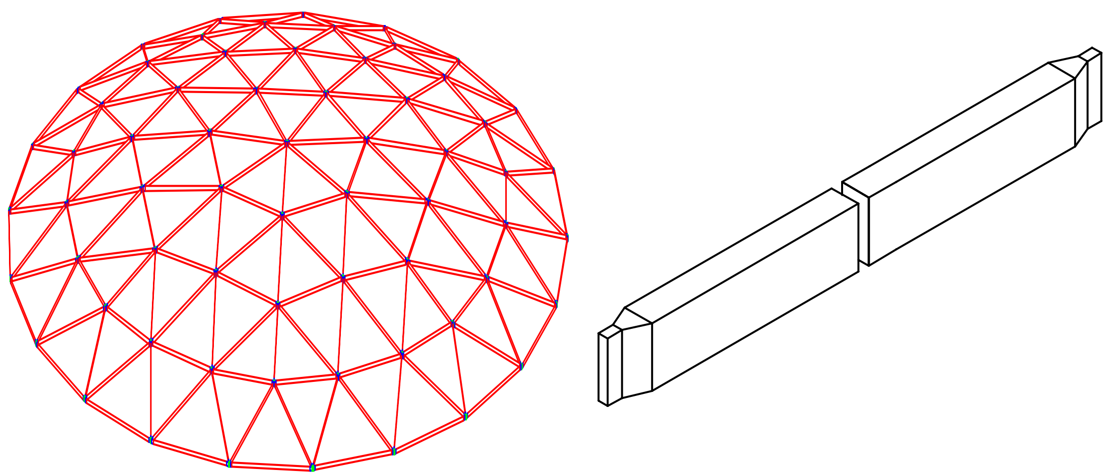

Figure 6: Spherical Dome Structure and Typical Beam Discretization

section and is again discretized using 5 plate elements. The size of individual elements depend upon the total length of each beam. It is assumed that the center part of the beam takes up 94% of the total length. The remaining 3% on each side of the beam are divided into 1.2% for the end part and the remaining 1.8% constitute the transition element, see Figure 6. The center and the two outer most beam sections have a constant thickness, while the remaining two are transition elements with a linearly varying thickness in the longitudinal direction. The center part is equal for all members and has a cross sectional dimension of 0.0454 x 0.1 meters. The thickness of the two end sections is reduced by decrements of 10%. Rectangular cross sections are used again in place of round tube elements commonly used in the “Triodetic” connector and in place of symmetrical “I” sections used for the Temcor joint system. Finally, all degrees of freedom are restrained at the support nodes except rotation around the vertical axis.

The modified Riks method is used to apply proportional loading and the force-deflection response is evaluated to determine different buckling loads and different failure modes. In Figure 7 the total applied load is shown as a function of the vertical displacement at the center node. As described above, the ratio of the thickness between the end parts and the center part is reduced by decrements of 10%, thus Stiffness = 100% signifies no change in stiffness (thickness) along the beam direction, while Stiffness = 12.5% means a reduction of beam thickness by a factor of 2. The sensitivity of the buckling load and/or failure mode from the in-plane bending stiffness can be examined from these graphs. Three different failure modes can be observed. First, for an end-section stiffness of 34.3% (70% of original thickness) and higher, the dome fails suddenly due to snap through buckling of a limited number of nodes, see Figure 8. The buckling load is essentially constant and above 3600 kN. Then, a transition zone can be observed where the failure mode is a combination of buckling and joint failure. In this transition, the ultimate buckling load can be considered constant as well with a magnitude of about 3500 kN. Finally the third failure mode is

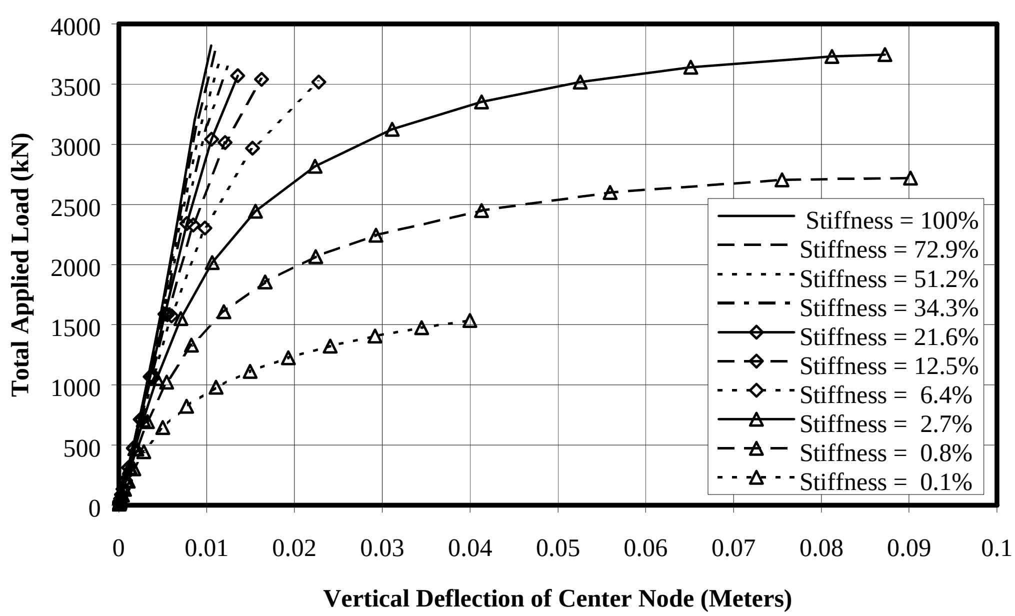

Figure 7: Sensitivity of Buckling and Failure Mode from Degree of Fixity

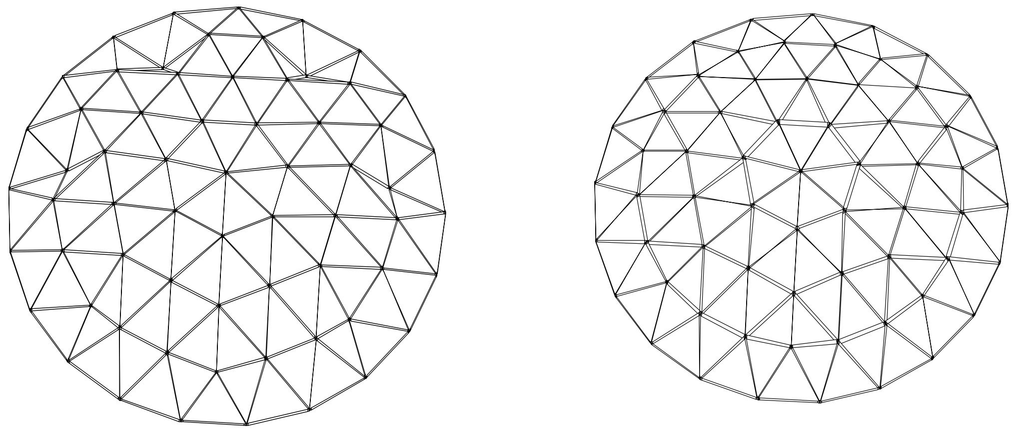

Figure 8: Failure Modes with End-Section Stiffness of 100% and 2.7%.

entirely due to failure of joints. For an end-section stiffness of 2.7% (thickness of 30%) and lower, no bifurcation buckling behavior does occur and the structure turns into a problem with continuos response with large nodal displacements and large rotations. The buckling load reduces as well consistent with the observation made in the previous section The deformed dome with a remaining in-plane bending stiffness of 2.7% is shown in Figure 8. Thus, it can be concluded that there exists a critical in-plane bending stiffness below which the load carrying capacity reduces proportionally to the reduction in rotational joint rigidity.

To evaluate the effect of the length of the transition section on the buckling response, the spherical dome is analyzed again. The same approximation of rectangular cross section and 5 plate elements for each beam member is made. The length of the center part of each beam is reduced to 88% of the original length and the 6% on each side of the beam are divided into 2.4% for the end part and 3.6% for the transition section, see Figure 6. The center and

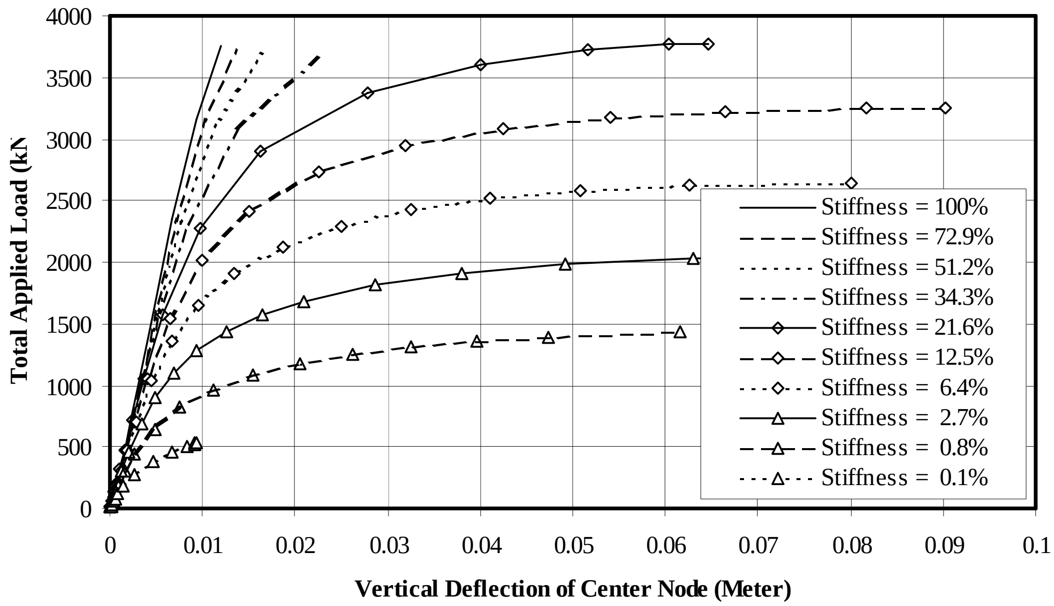

Figure 9: Load-Deflection Response with an Increased Length of Transition Section

the two end-sections have constant cross-section, the thickness of the transition element varies linearly along the beam direction. The load-deflection response is shown in Figure 9. Again, different failure modes can be observed depending on the amount of bending resistance at the joints. The increase of the transition section reduces the joint rigidity for in-plane rotation and compared to the previous example, a reduction in the in-plane bending stiffness to 34% is sufficient to change from a buckling mode into the joint failure mode. Thus, it may be concluded that the failure mode is not only reduced by the in-plane bending resistance at the joint connection but also by the length of the transition element.

CONCLUSIONS

A numerical approach was used to evaluate the influence of rotational joint stiffness on the bifurcation buckling response of reticulated dome structures. Each beam element is approximated using rectangular cross section elements and is discretized using five plate elements aligned along the main axis of the beams. First, a simplified structure is used to show that the reduction in load carrying capacity reduces proportionally with the joint in-plane bending stiffness. Then, a spherical dome with a diameter of 24 meters was analyzed to establish the effect of joint stiffness on the load carrying capacity of spherical domes. For the dome model, it was found that as the joint stiffness is reduced, the dome failure mode changes from a local snap through buckling mode into a problem with continuous response with large nodal displacements and large rotations. For this second failure mode, the load carrying capacity depends directly from the joint stiffness. For each reduction in joint stiffness, a corresponding lower buckling load was obtained.

Thus, it may be concluded that a critical joint stiffness exists above which the buckling load can be assumed to be independent of the connector's rotational stiffness characteristics. On the other hand, for joints with a degree of fixity lower than the critical value, the load carrying capacity reduces proportionally to the joint stiffness. For systems with known high rotational stiffness, such as the double plate hub system referenced in this study, joint rotation need not be a concern for the practitioner. When designing dome systems using joints with low rotational stiffness, such as the Triodetic joint, however, careful consideration of the reduction in buckling strength should be taken into account.

REFERENCES

[1] ABAQUS/Standard, Version 5.7, User's Manual, Hibbitt, Karlson & Sorensen, INC. Pawtucket, RI., 1998.

[2] Buchert, K.P. 1969. Local and General Buckling of Reticulated Shells. *International Association for Shell Structures.* International Colloquium, Madrid, Spain.

[3] Buchert, K.P. 1973. *Buckling of Shell, Shell-Like Structures.* Columbia, Missouri, USA.

[4] Brush, D.O., Almroth, B.O. 1975. Buckling of Bars, Plates and Shells. *McGraw Hill*, New York.

[5] Bushnell, D. 1985. Computerized buckling analysis of shells. Martinus Nijhoff Publishers, Kluwer Publishers Group.

[6] Crooker, O.J., Buchert, P.K. 1970. Reticulated Space Structures. *Journal of the Structural Division,* ASCE, Vol. 96, No. ST3: 687-700.

[7] Elliott, A. W., 1984. Triodetic Domes, *Analysis, Design and Construction of Braced Domes.* Edited by Z.S. Makowski. Univeristy Press, Cambridge, 1984.

[8] Fujimoto, M. et al. 1993. Experimental and Numerical Analysis of the Buckling of a Single-Layer Latticed Dome. *Proceedings of the 4th International Conference on Space Structures.* Surrey, U.K., 1993.

[9] Gioncu, V., Balut, N. Instability Behavior of Single-Layer Reticulated Shells. *International Journal of Space Structures.* Vol. 7, No. 4: 243-252.

[10] Hanaor, A. 1995. Characteristics of Prefabricated Spatial Fame Systems. *International Journal of Space Structures,* Vol. 10, No. 3, 1995: 151-173.

[11] Holzer, K. S. et al. 1980. Stability of lattice Structures under Combined Loads. *Journal of the Engineering Mechanics Division,* ASCE, Vol. 106, No EM2: 289-305.

[12] Ikeda, K., Torii, K. 1987. Group Theoretic Description of Bifurcation Behavior of Axisymmetric Regular-Polygonal Truss Domes. *Structural Eng./Earthquake Eng.* Vol. 4. No. 2: 237-247.

[13] Kani, M.I., McConnel E.R. 1988. Collapse of Shallow Lattice Domes. *Journal of Structural Engineering,* ASCE, Vol. 113, No. 8: 1806-1819.
PVP, Vol. 176:67-74.

[14] Lopez, E.A., Orrison, K.G. 1994. Design of Low Rise Dome for University of Hawaii at Manoa. *Proceeding of the IASS-ASCE International Symposium 1994 on Spatial, Lattice and Tension Structures.*

[15] Lopez, E.A., Orrison, K.G. 1995. Critical Buckling Loads of Triangulated Dome Structures with Pinned Connections. *Numerical methods Structural Mechanics.* AMD-Vol. 204.

[16] Lopez, E.A., Orrison, K.G. 1995. The Effect of Joint Stiffness on the Buckling Behavior of Reticulated Dome Structures.

[17] Lopez, E.A., Troup, K. 1996. Aluminum Lattice Structures: Developments and Innovations ion Clear Span Roof Solutions. *International Symposium on Conceptual Design of Structures.* Stuttgart, Germany.

[18] Mang, H.A., Helnwein, P. 1995. A Priori Estimates of Stability Limits on Nonlinear Load- Displacement Paths. *Proceedings of the International Conference on Lightweight Structures in Civil Engineering.* Warschau, Polen. Ed.: J.B. Obrebski, Magat Warschau, Vol. II: 811-821.

[19] Meek, J.L., Loganathan, S. 1989. Theoretical and Experimental Investigation of a Shallow Geodesic Dome. *International Journal of Space Structures,* Vol. 4, No. 2: 85-105.

[20] Schlaich, J., Schober, H. 1997. Glass Roof for the Hippo House at the Berlin Zoo. *Structural Engineering International.* 4/97:252-254.

[21] See T., McConnel E.R. 1986. Large Displacement Elastic Buckling of Space Structures. *Journal of Structural Engineering,* ASCE, Vol. 112, No 5: 1052-1069.

[22] Sugizaki, K., Kohmura, S. 1994. Experimental Study on Buckling Behavior of a Triodetic Aluminum Space Frame. *Proceedings of the IASS-ASCE International Symposium 1994.* April 24-28, 1994, Atlanta, Georgia: 478-487.