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Basic concepts and analysis of a new cable hanging roofs

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Abstract A new light weight self-contained cable-space truss composite roof structure has been presented. The cable system cannot only stiffen the structure but also facilitate the construction process, even more, it takes no more additional spaces around the structure. Algorithm for the analysis and design of the cable system has been presented. Some numerical examples have been given also.

Key Words space truss, cable, roof

1 Introduction

Cables made from wires with high tensile strength are widely used in large span structures like bridges and roof structures[1-5]. There are many forms and combinations in the structural configuration for different purposes. Usually, bulk anchoring parts are needed to withstand the huge tensile force from the cable systems and which may take much additional spaces around the structures.

This paper presents a new cable hanging roof structure, which uses the space truss as the primary structure and make use of the cable system as an additional part to stiff the structure as well as to facilitate the construction process. The configuration of the new composite structure is shown in figure 1.

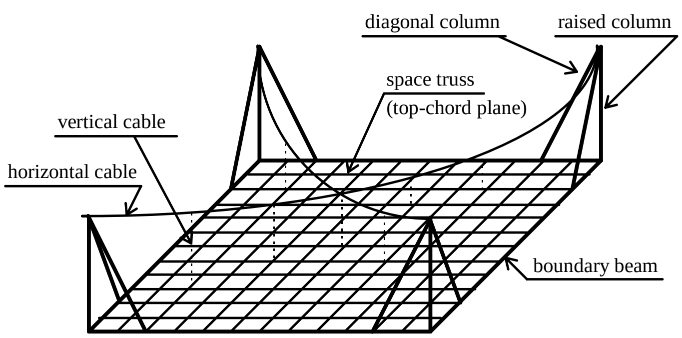

figure 1 draft of the structural configuration

The rectangular plate like space truss is supported by the boundary beam. The column at each of the four corners is raised and two diagonal columns are added to share the shear force. Two horizontal cables connect the top of the raised column in the cross form and a number of vertical cables is needed to connect the horizontal cable and the top-chord of the space truss to make up an additional load bearing cable system.

The basic assumptions for this structure include: (1)The space truss consists of ideal pin-joined straight bars made of linearly elastic material, all loads are acted at the nodes and the displacement and strains are small; (2)All of the cables are ideally flexible, the material is linearly elastic, the strain is small, the displacement is finite, the self-weight of the cable can be neglected.

2 Design of the horizontal cable

Suppose there are totally m vertical cables on one of the horizontal cables, the joint coordinates are (x_i,z_i) respectively, the tensile force in each of the vertical cable is Q_i under the normal working condition. The horizontal cable is divided into m+1 straight segment when assumption (2) is taken into consideration. Suppose the support of the horizontal cable is at the same level and the span is L (see figure 2).

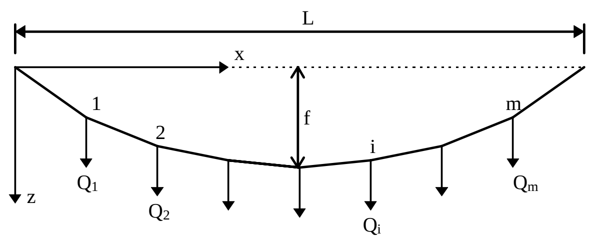

Figure 2 draft of the horizontal cable

The relationship between the vertical displacement $W(x)$ and the horizontal component H of the tensile force of the horizontal cable is known as

$$W(x) = M(x)/H \tag{1}$$

Where $M(x)$ is the moment of the corresponding simply supported beam with the same span and loads. When the mid-span moment M_c of the corresponding simply supported beam is known, the horizontal component H will be

$$H = M_c/f \tag{2}$$

Then the coordinate Z_i of the joint i of the cable will be

$$Z_i = M_i/H \tag{3}$$

Where M_i is the moment of the corresponding simply supported beam at point i

$$M_i = \frac{x_i}{2}\sum_{j=1}^{i-1} Q_j - \sum_{j=1}^{i-1}\left(x_i - x_j\right)Q_j \tag{4}$$

The length of each segment i of the cable is

$$S_i = \sqrt{\left(x_i - x_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2} \tag{5}$$

and the tensile force in each segment is

$$T_i = HS_i/\left(x_i - x_{i-1}\right) \tag{6}$$

Because the strain of the cable in each segment is

$$\varepsilon_i = \left(S_i - S_{i0}\right)/S_{i0}$$

the original length of each segment will be

$$S_{i0} = S_i\Bigg/\left(1 + \frac{T_i}{AE}\right) \tag{7}$$

and the original length of the horizontal cable is

$$S_0 = \sum_{i=1}^{m+1} S_{i0} = \sum_{i=1}^{m+1} S_i\Bigg/\left(1 + \frac{T_i}{AE}\right) \tag{8}$$

Where A and E are the cross sectional area of the cable and its elastic modulus. Equation (2) though (8) can be used to design the horizontal cables.

3 Design of the vertical cable

In engineering practice, the parameters of f and H of the horizontal cables are usually prescribed at first. The effectiveness of the cable system is determined by the number, location and tensile force (or length) of the vertical cables.

The objective at present is to reduce the maximum internal forces of the space truss and even the distribution of the internal force of the space truss with little additional cables. Suppose there are m vertical cables and the control set contains n bar elements which is taken from the space truss for some special purposes. Let the tensile force be 1 in jth vertical cable and 0 in others, while the change of the internal force in the ith bar of the space truss is a_{ij}, a_{ij} is called the internal force impact factor of cable j to bar i of the space truss. The ultimate force of the bars of the space truss will be

$$N_{i} = N_{i}^{P} + \sum_{j=1}^{m} a_{ij} Q_{j} \tag{9}$$

Where N_i^P denotes the internal force of bar i under normal loads which can be determined by the FEM, $Q_j \geq 0$ is the working tensile force of cable j.

It is quite evident that along with the increase of Q_j the absolute value of N_i will decrease when a_{ij} and N_i^P take different signs. Let g_j be the internal force decrease factor of unit tensile force of cable j to a certain control bar set,

$$g_{j} = \sum_{i=1}^{n} \mathrm{sign}\left(N_{i}^{P}\right) \times a_{ij} \tag{10}$$

Let b_j denotes the relative internal force decrease factor due to the unit tensile force of cable j,

$$\begin{aligned} b_{j} &= g_{j} / g_{1} \quad , \quad 2 \leq j \leq m \\ b_{1} &= 1.0 \end{aligned} \tag{11}$$

It is evident that the value of b_j(j=1,…,m) can serve as the optimum distribution of the relative tensile force factor of the vertical cables. For the given f and H, taking equation (2,4) into consideration, the working force factor F for all of the vertical cables will be

$$F = Hf \Big/ \sum_{j=1}^{m1} b_{j} x_{j} \tag{12}$$

Where m1 denotes the total number of the vertical cables in the half span of the horizontal cable, x_j denotes the horizontal distance of cable j to the mid-span. The tensile force Q_j of the vertical cable i under the normal working condition will be

$$Q_{j} = F b_{j} \tag{13}$$

The length B_j of vertical cable j is known as

$$B_{j} = H_{c} - z_{j} + W_{j} \tag{14}$$

Where H_c denotes the length of the raised column, W_j (got by the FEM) denotes the vertical displacement of the space truss at connection point j, z_j is known from equation (3). Because the strain of cable j is

$$\varepsilon_{j} = \frac{B_{j} - B_{j0}}{B_{j0}} = \frac{Q_{j}}{EA_{j}}$$

while the original length of the vertical cable j will be

$$B_{j0} = B_{j} \Big/ \left(1 + \frac{Q_{j}}{EA_{j}}\right) \tag{15}$$

Where A_j denotes the cross sectional area of cable j.

The calculation and design of the raised column and diagonal column is easier to do when the tensile force and the shape of the horizontal cable is known. The calculation and design of the new cable hanging composite roof will easily be done using equation (2) through (15). Some numerical examples will be given in the next section.

4 Numerical examples

Example 1. An orthogonal square pyramid space truss measures 36m×36m with a grid dimension of 3m×3m and 2.4m in depth. The space truss is supported by the boundary beam and subjects to 1KN/M^2 uniform loads. The cable system presented in the above section is added into this structure. Suppose the basic parameters are H=100KN (needs approximately one cable of 7ϕ5) and f =5m. Only one quarter of the structure is needed to consider for the symmetry of the structure. The results are shown in table 1 and table 2.

Table 1 relative tensile force factor of the vertical cables

control bar set	b_1	b_2	b_3	b_4	b_5	b_6	F
top chord	1.0000	3.0723	5.3956	7.4863	8.9372	4.7286	0.9329
bottom chord	1.0000	2.9644	5.2958	7.5407	9.2619	5.0672	0.9087
diagonal chord	1.0000	2.0987	3.1582	4.2129	5.2661	3.1594	1.5343
all bars	1.0000	2.7493	4.7080	6.5542	7.9893	4.3972	1.0414
max. compression	1.0000	3.7594	7.7838	12.8534	20.8759	13.0583	0.4430
max. tension	1.0000	3.7130	7.6453	12.6856	19.7038	18.2304	0.4064

notes: the figure in the column b_6 is only one quarter of its real value for the symmetry.

Table 2 sum of the absolute internal force of the space truss decreased (%)

cont. bar set	top chord	bott. chord	diag. chord	all bars	max. comp.	max. tens.
top chord	13.997	13.659	13.801	15.285	17.464	16.501
bott. chord	13.921	13.577	13.729	15.214	17.512	16.506
diag. chord	13.753	13.403	13.549	15.092	17.317	16.240
all bars	13.917	13.574	13.721	15.220	17.455	16.452
max. comp.	13.459	13.086	13.305	14.714	17.901	16.637
max. tens.	12.729	12.359	12.598	13.924	17.675	15.975

It is shown in table 1 that the tensile force factors in the vertical cables increase from the edge to the mid-span but the distribution of the values is different from one control bar sets to the other. According to the tensile force factor and the working force factor for different control bar sets listed in table 1, the decrease of the sum absolute internal force of different bar sets in the space truss are listed in table 2. Because the difference of the values in the same column of table 2 is very small, any control bar set can be taken to determine the tensile force distribution of the vertical cables.

When the top chord set is taken as the control bar set, the value of the total bars' sum absolute internal force decreases most, so this set can be taken as the key control bar set. As the decrease of the sum absolute internal force is approximately 15% in this case, it cab be said that the total steel consumption decreases about 15% for the steel consumption is proportional to the internal forces. Because the steel spend on the cable system is very small while the saving of the steel on the space truss is comparatively very large, the economical value of the new structure is evident.

Besides used as part of the composite structure, the cable system can also be used as the temporary supports for the assembling or erection of the space truss. The feature of self-contain which means that the composite structure don't need additional anchoring system and don't take additional spaces around the structure is very important when the roof structure is located in a very compact area or the pillars which support the structure are very high.

Example 2. The conditions are the same as in example 1 except that *H*=200KN. The top chord set is taken as the control bar set. The results are shown in table 3.

Table 3 sum of the absolute internal force of the space truss decreased (%)

cont. bar set	top chord	bott. chord	diag. chord	all bars	max. comp.	max. tens.
top chord	27.656	27.317	27.601	28.049	34.615	33.002

Example 3. The conditions are the same as in example 1 except that *f* =8m. The top chord set is taken as the control set. The results are shown in table 4.

Table 4 sum of the absolute internal force of the space truss decreased (%)

cont. bar set	top chord	bott. chord	diag. chord	all bars	max. comp.	max. tens.
top chord	22.174	21.854	22.081	23.159	27.942	26.402

Table 3 and table 4 show that the internal force of the space truss or the steel consumption of the space truss decreases along with the increase of the basic parameter of *f* or *H* of the horizontal cables. It is not difficult to find that the variation is linearly related from equation (9-13) or tables 1-4. The nonlinear feature of the horizontal cable is sheltered because the interaction of the cable system and the space truss are calculated separately and the tensile force and shape of the horizontal cable are determined according to the optimal tensile force distribution of the vertical cables in advance. This results in a set of simple formulations and lead an easy way for the engineering practice besides its economical and space saving merits.

5 Conclusions

The new self-contained cable-space truss composite structure presented in this paper has the merits of economy, space saving, fit for tall buildings and easy to construct. The formulations are easy to use and the design process is simple. But the research work is only at its primary stage, the dynamic character of the new structure is left to be studied and the engineering practice is left to be done.

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