ANALYSIS OF THE CRITICAL STATE OF DOUBLE-LAYER SPACE GRIDS FITTED TO A FLAT SURFACE

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ABSTRACT

In the paper a procedure is presented to analyse pin-jointed double-layer space frames fitted to a flat surface. The change of state of the structure under continuously increased one-parameter load of arbitrary arrangement is described up to the first occurrence of buckling or braking of bars. The force in the buckled bars, with good approximation, is considered constant, and the load can be increased further, while additional bars buckle or break. The redundancy, and so the whole stiffness, of the structure decreases with the increase of the number of the buckled or broken bars. The one-parameter load can be increased until the structures will be statically overdeterminate. It means the collapse of the space frame.

MODEL OF THE GRID

The trihedron plays a significant role in the numerical description of the assembly [3]. A trihedron is defined by three non-coplanar bars having one end point in common that is considered as starting point of the bars in the trihedron. In Fig. 2, bars in a trihedron at each node are shown by arrows pointing outwards from the node. The assembly built up from these trihedron bars does not constitute necessarily a statically and kinematically determinate structure (Fig 1). The “number of bars multiplied by three equals the number of internal nodes” is a necessary but not sufficient condition of the statical and kinematical determinacy. This can be decided by the determination of the rank of the equilibrium matrix.

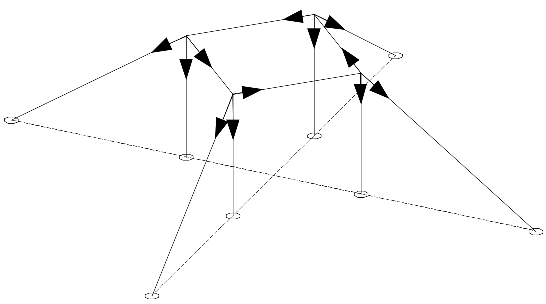

Figure 1.

The structure in Fig. 2 has 24 internal nodes, 13 external nodes, 3·24=72 trihedron bars and 16 additional bars. The additional 16 bars (thick lines in Fig. 2) in the structure are redundant. Therefore, the model in the unloaded state is at least 16 times statically indeterminate.

The equilibrium matrix of the assembly can be partitioned as:

$$\mathbf{G}=\mathbf{G}_{11}+\mathbf{G}_{12} \tag{1}$$

$\mathbf{G}_{11}$ is the non-singular equilibrium matrix of the subassembly composed of the trihedron bars, $\mathbf{G}_{12}$ includes the redundant bars. The equilibrium equation of the grid is:

$$\mathbf{G}\bullet\mathbf{s}+\mathbf{q}=0 \tag{2}$$

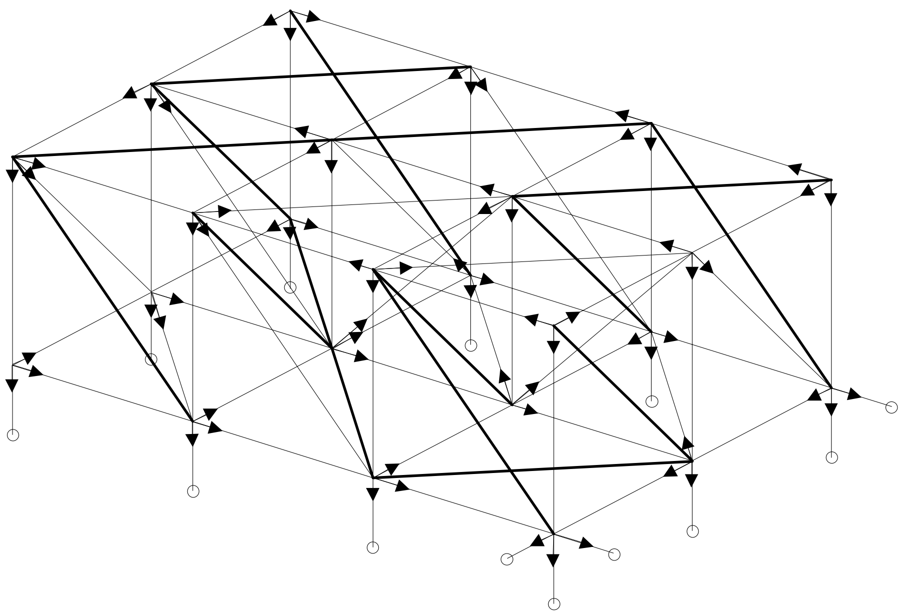

Figure 2.

We need the inverse of submatrix $\mathbf{G}_{11}$:

$$\mathbf{Q}_{11}=\mathbf{G}_{11}^{-1} \tag{3}$$

This can be made without filling the whole matrix $\mathbf{G}_{11}$. First the hyperdiagonal of $\mathbf{Q}_{11}$ is produced with the help of the unit vectors of bars.

$$\mathbf{Q}_{11}^{<0>}=\begin{bmatrix} \begin{bmatrix} e_{1,1_x} & e_{1,2_x} & e_{1,3_x} \\ e_{1,1_y} & e_{1,2_y} & e_{1,3_y} \\ e_{1,1_z} & e_{1,2_z} & e_{1,3_z} \end{bmatrix}^{-1} & & & \\ & \begin{bmatrix} e_{2,1_x} & e_{2,2_x} & e_{2,3_x} \\ e_{2,1_y} & e_{2,2_y} & e_{2,3_y} \\ e_{2,1_z} & e_{2,2_z} & e_{2,3_z} \end{bmatrix}^{-1} & & \\ & & \ddots & \\ & & & \begin{bmatrix} e_{24,1_x} & e_{24,2_x} & e_{24,3_x} \\ e_{24,1_y} & e_{24,2_y} & e_{24,3_y} \\ e_{24,1_z} & e_{24,2_z} & e_{24,3_z} \end{bmatrix}^{-1} \end{bmatrix}$$

The first 3×3 submatrix is the inverse of that of the unit vectors of trihedron bars associated to the first internal node, the second submatrix associated to the second internal join can be made in the same way as the previous submatrix, etc. With this technique the positive unit vectors in $\mathbf{G}_{11}$ are built into the inverse matrix. The negative unit vectors of bars connecting internal joints are taken into account barwise by the Sherman-Morrison formula [1]. Proceeding, we produce the inverse of the equilibrium matrix supplemented by a dyad

$$\left(\mathbf{G}_{11} + \mathbf{f} \bullet \mathbf{h}^T\right)^{-1} = \mathbf{Q}_{11} - \mathbf{Q}_{11} \bullet \mathbf{f} \bullet \mathbf{h}^T \bullet \mathbf{Q}_{11} / d\,, \tag{4}$$

where

$$\mathbf{Q}_{11} = \mathbf{G}_{11}^{-1}, \qquad d = 1 + \mathbf{h}^T \bullet \mathbf{Q}_{11} \bullet \mathbf{f} \neq 0\,. \tag{5}$$

In the case of the *r*th trihedron bar, if we denote its end point by *j* (internal node), then in the *j*th subvector of column vector **f**, is the unit vector of *r*th bar multiplied by –1. The *r*th element of row vector $\mathbf{h}^{\mathrm{T}}$ is 1, the other elements are 0. If d=0 then the matrix **G** cannot be supplemented by the next dyad, therefore the rank of **Q** (or rather of $(\mathbf{G} + \mathbf{f} \bullet \mathbf{h}^{\mathrm{T}})$) decreases by 1.

THE PROCESS OF LOADING

At the internal nodes, arbitrary one-parameter load can be applied. In the example, at the nodes of the upper layer, we used uniform vertical forces pointing downwards. If force in bar attains the buckling force, then the bar will buckle. The buckled bar is replaced with its constant buckling force, applied at the two end points, acting along the straight line connecting the end points of the bar. This force is considered as external load which is kept constant until the distance between the end points of the buckled bar reaches again the length of the bar prior to buckling. These "external" buckling forces do not change during the increase of the one-parameter load.

If the buckled or broken bar is a redundant bar, then the trihedron bars are unchanged. On the other hand, if a trihedron bar is buckled or broken, then we supplement the incomplete trihedron so that we replace the missing bar with the nearest redundant bar directly or with the help of a chain of rearranged trihedra leading to the nearest redundant bar (Fig. 3). The matrix $\mathbf{Q}_{11}$ changes in one step with the addition of a hyperdyad:

$$\left(\mathbf{G}_{11} + \mathbf{F} \bullet \mathbf{H}\right)^{-1} = \mathbf{Q}_{11} - \mathbf{Q}_{11} \bullet \mathbf{F} \bullet \mathbf{D}^{-1} \bullet \mathbf{H} \bullet \mathbf{Q}_{11}$$

where

$$\mathbf{Q}_{11} = \mathbf{G}_{11}^{-1}; \quad \mathbf{D} = \mathbf{E} + \mathbf{H} \bullet \mathbf{Q}_{11} \bullet \mathbf{F}; \quad \det(\mathbf{G}_{11}) \neq 0; \quad \det(\mathbf{D}) \neq 0\,.$$

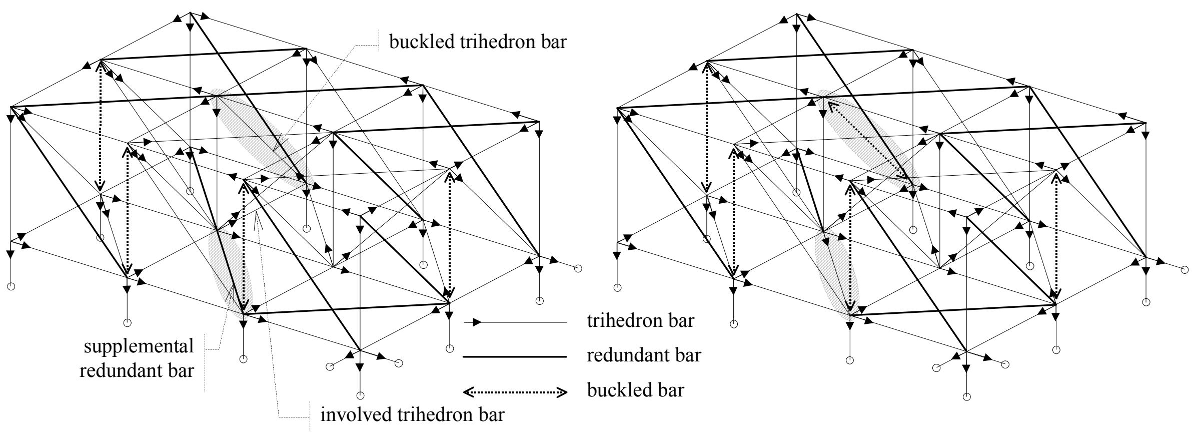

Figure 3.

Number of rows of **H** and number of columns of **F** are identical to the number of trihedra involved with rearrangement.
The redundancy is decreased if a bar is buckling whether the buckled bar was a trihedron or a redundant bar. The process of loading can be continued until the next buckling or breaking. The process is stopped if there are no more redundant bars, or with all redundant bars det(**D**)=0. Then the grid is statically overdeterminate, the grid is working as a mechanism. The advantage of the algorithm is that the last inverse matrix $\mathbf{Q}_{11}$ can be used also in the analysis of mechanisms (finite displacements of kinematically indeterminate bar structures) [6][7].

CONCLUSION

The equation matrix of the trihedron bars can modify simply due to the Sherman-Morrison formula [1]. With tracing the change of state we can study the behaviour of the assembly. By arbitrary load pattern it can be investigate the assignment of the bars, configuration of the grids, where must be adding more bars, where expedient to increase the stiffness of bars [4].

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