Tensile Structures Shape Definition Using Digitalised Topometric Model Research

Zdenko Jecić, B. Eng.
Faculty of Architecture, University of Zagreb
Kaciceva 26,
10000 Zagreb, Croatia

phone +385 1 332 804
fax +385 1 482 80 79
E-mail: zdenkojecic@hotmail.com

1. Abstract

The integration possibilities of architectural models in the design process of the tensile structures have been investigated. The 3D shape of two physical models has been measured with precise optical method, based on triangulation principle and digital image analysis. The obtained results have been compared with numerical methods and a good agreement has been achieved. This way, the optically digitized shapes of models are suitable for further computer visualization, numerical form-finding and structure optimization procedure.

2. Introduction

Tensile structures like cable-nets and textile membranes are in recent years, because of their lightness and technological features, very often used for roof covering of permanent and temporary wide-span buildings (stadiums, exhibition and sport halls). Since these structures are exclusively tensile resistant, they are characterized by self-forming process, where the form of the structure can be influenced only by changing the boundary conditions and prestressing forces. This is the reason why the form-finding of the structure appears as the special engineering problem, where satisfying conditions of equilibrium and structural safety, as well as the architect's intentions, shape of the structure is to be determined [2]. This procedure usually involves solving of highly non-linear equations of equilibrium that requires starting approximation. Applying linearised equations, the force-density method is the most convenient for numerical approximation of the shape.

In the first stage of the tensile structures design process, where architects are developing their basic idea, they have to estimate the shape of the structure. Their estimation must be as near as possible to the shape that can be achieved by form-finding process. Dealing with this problem, architects usually use simple physical models, which always satisfy equilibrium conditions and offer excelled visualization (Figure 1). Since experience and ”feeling” for the structure are crucial for the good estimation, such models are very important in architect education.

In order to increase the efficiency of the design process, there is present requirement of integrating its components [3], [6]. The main idea of this work is to investigate possibilities of architectural models as the basic approximation for the further numerical form-finding process, which is usually done by means of computer (Figure 2). For this purpose it is needed to determine the 3D geometry of the model, where digitized output results are recommended. In this work automated digital topometric sensor Atos is used for 3D optical shape determination of two types of convenient architectural models.

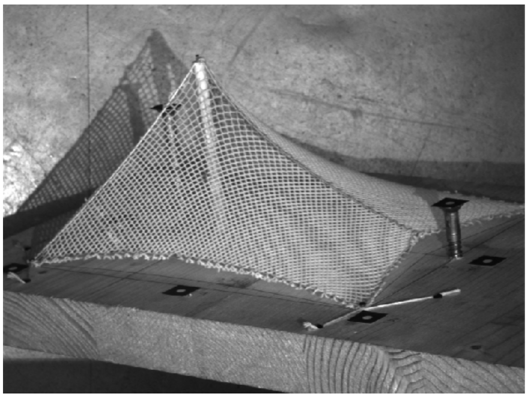

Figure 1.
Architectural model of a membrane roof

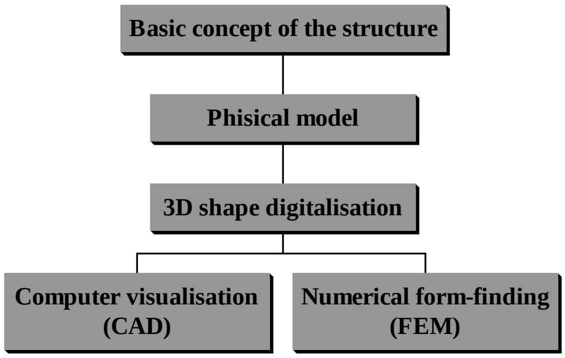

Figure 2.
Proposal for the improved design process

3. Procedure

The method is tested on simple boundary condition task of minimal net (such configuration where the whole cable length is minimal). Minimal net is spanned over 24 ∗ 16 m area with 4 m distance between XY coordinates of cable end points. Midpoints of longer sides are risen 9 m (Figure 3). It is supposed that there is no friction between cables, and that all cables are uniquely prestressed at their ends. In such case, forces along cables in every part of the structure will be the same, and cables will come into position where their total length will be minimal (in such case the cables form minimal net).

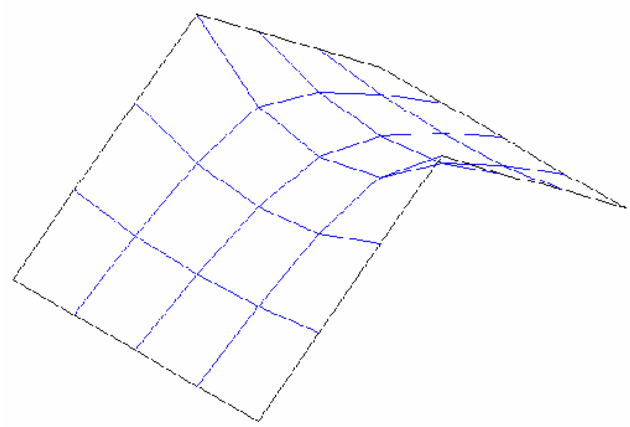

Figure 3. Task of minimal net

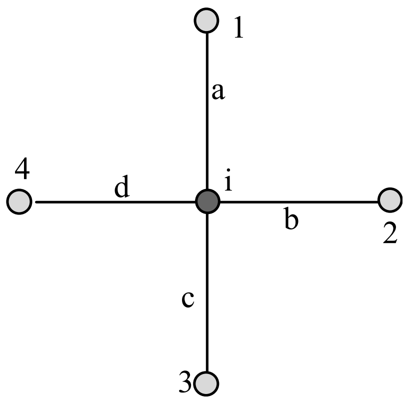

Figure 4. i knot of net

Equations of static equilibrium for each knot of the structure (points where two cables are intersecting) according to Figure 4, and supposing that the structure is not loaded except with unique prestressing forces are [1]:

$$\sum_{j=1}^{4} \frac{\left(x_i - x_j\right)}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}} S_{ij} = 0 \text{........(2.1.)}$$

$$\sum_{j=1}^{4} \frac{(y_i - y_j)}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}} S_{ij} = 0 \text{ .... (2.2.)}$$

$$\sum_{j=1}^{4} \frac{(z_i - z_j)}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}} S_{ij} = 0 \text{ .... (2.3.)}$$

Since these equations are non-linear, numerical solving requires an approximation as the starting point. The most common numerical method for approximation is force-density method [5] where equations 2.1. - 2.3. are linearised introducing:

$$q_{ij} = \frac{S_{ij}}{l_{ij}}, \text{ .... (2.4.)}$$

For such case, equations 2.1. – 2.3. are transformed into:

$$(q_a + q_b + q_c + q_d)\, x_i - q_a\, x_1 - q_b\, x_2 - q_c\, x_3 - q_d\, x_4 = 0 \text{ .... (2.5.)}$$

$$(q_a + q_b + q_c + q_d)\, y_i - q_a\, y_1 - q_b\, y_2 - q_c\, y_3 - q_d\, y_4 = 0 \text{ .... (2.6.)}$$

$$(q_a + q_b + q_c + q_d)\, z_i - q_a\, z_1 - q_b\, z_2 - q_c\, z_3 - q_d\, z_4 = 0 \text{ ....(2.7.)}$$

Solutions of these equations are easily found if q_{ij} is supposed to be constant. If such approximation does not satisfy, new q_{ij} can be calculated and the procedure repeated.

To evaluate the experimental results, they are compared with the results achieved by force-density method improved in one iteration, and final shape of the minimal net achieved by equations 2.1. – 2.3. First values of coordinates are determined according to 2.5. – 2.7. and q_{ij} = const. = 0,25. In next step new q_{ij} are calculated for each element supposing that Sij = const., and improved coordinates were achieved. Finally, these approximate results were introduced as the starting point for numerical solving of equations 2.1. – 2.3. and the shape of minimal net was defined. Results of force-density method as well as the final results are shown in Figure 5.

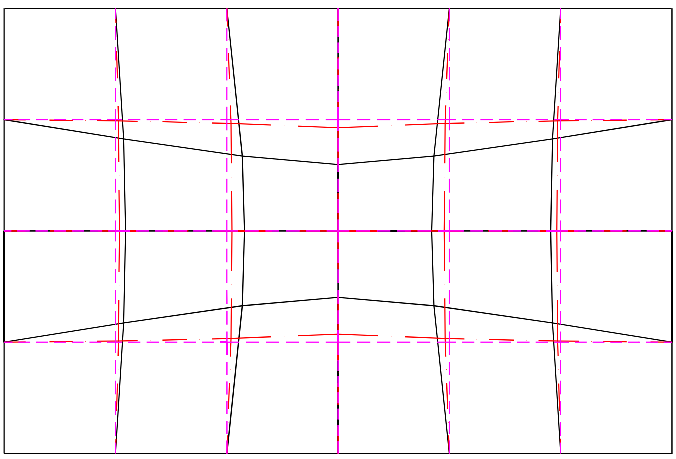

Figure 5. Cable position in XY plane: ——— minimal net, - - - force-density method, —— —— improved force-density method

4. Experimental determination of approximate shape

Selection of type of the model and the measuring procedure was made regarding to following basic requirements:

- Model building technology has to be simple and economic.
- Models and measuring procedure have to ensure satisfying accuracy of shape approximation for further non-linear numerical analysis.
- Measuring procedure has to be automated with digitized output results.

Models

As the main goal of this work is to prove the possibilities of use of architectural models in further stages of form-finding process, chosen were commonly used models, which can be built in very short period of time and with minimum of necessary equipment. Geometric similarity has to be ensured as well as the equality of prestressing forces and their distribution among elements.

On described task two types of models were tested. In both cases, boundaries were presented by frame made of wooden sticks, with maximal error of ± 1.5%. Model 1 (Figure 6) is more similar to the physical reality. On frame were freely hanged threads equally loaded on their ends. Model 2 (Figure 7) was made by stretching thin elastic fabric with orthogonal mash over boundary frame. This type of model is more often used for architectural purposes. Controlling the size of mash, it was tried to ensure the equal prestressing force distribution. This type of model is more similar to the idealized shape of minimal surface, but for simple tasks it can also represent minimal net [4].

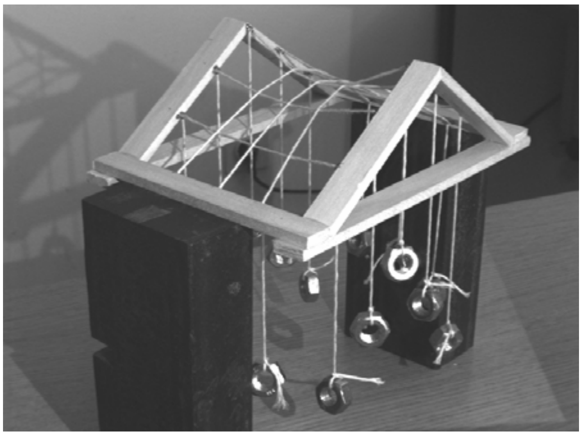

Figure 6. Model 1

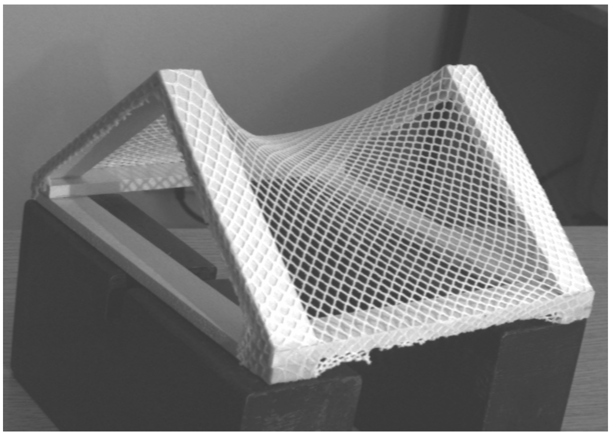

Figure 7. Model 2

Measuring procedure

Basic intentions of this work required an easy in use and economic measuring procedure with digitized output data. Therefore it was chosen non-contact optical topometric method by using measuring system Atos (Advanced Topometric Sensor) shown on Figure 8, and produced by GOM company (Braunschweig, Germany). This system has wide range of possible applications in 3D shape digitalization (Figure 9).

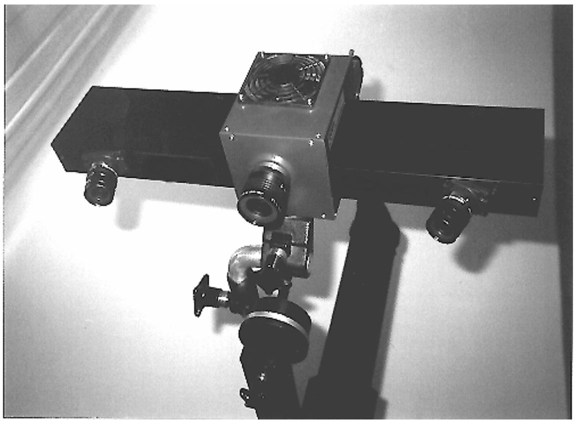

Figure 8. 3D topometric sensor Atos *Figure 9.*

The components of this system are: central measuring device - consisting of a special projector and two CCD cameras, control device, analogue-digital converter, computer and specialized software. Instead of using fixed measuring points (like in stereophotogrammetric measurements), this sensor projects various fringe pattern onto the object surface, which are than recorded by two cameras from different angles (Figure 10). Using triangulation method and digital image processing, precise 3D coordinates for each of the 439,296 camera pixels are independently calculated, creating so called "cloud" (Figure 11). Simultaneous calibration during measurement helps to reach an accuracy better than ± 0.05 mm for the measurement volume of 200∗120∗160 mm. By changing its geometry and because of fast calibration routines, this sensor can be easily fitted to other object sizes.

Measuring procedure is highly automated and simple handling, where output results are available in few seconds. 3D coordinates of object surface points are presented as a text-file, and therefore suitable for further CAD or FEM computing techniques.

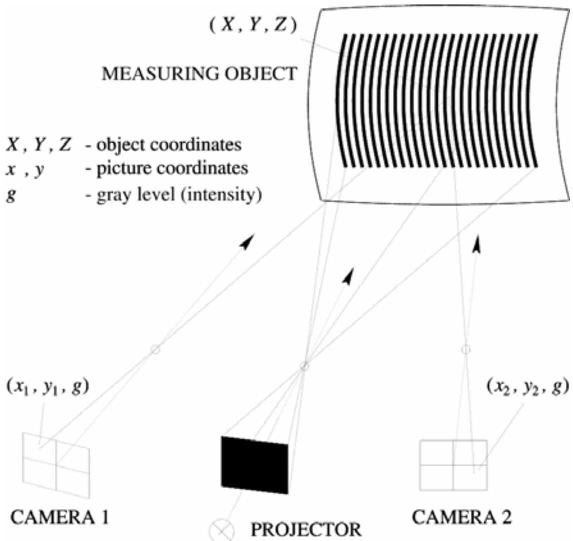

Figure 10. Measuring principles

For model 1 number of measured points was 32432 and for model 2 was 248 618, what was reduced to 364 and 3181 points. Selection of knot points, which are comparable with numerically calculated ones, was by model 1 done directly, using system software (Figure 12). After transformation of coordinate system, final 3D coordinates of points where two threads are intersecting each other were achieved.

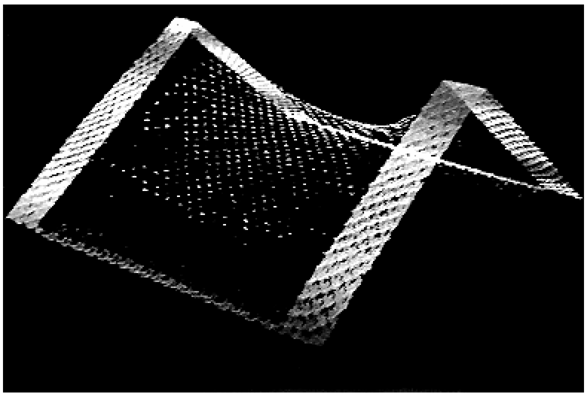

Figure 11. Cloud of measuring points (model 2)

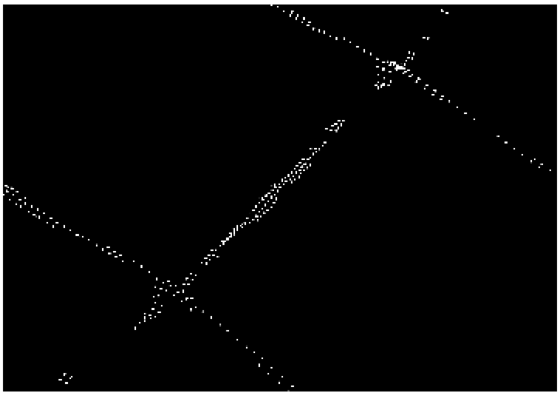

Figure 12. Net knots of model 1

Since model 2 has no characteristic points analogue to the calculate minimal net knots, chosen were points of XY plane orthogonal grid (4 x 4m) and their Z coordinates were measured. This results are comparable with minimal net approximation using first step of force –density method. Due to the model 2 elastic fiber discontinuity (mash size of max. 2,4 mm) an error of XY plane point location was max. ±1,2 mm. In order to compare the shape of the model 2 surface with the idealized minimal net, Z coordinate difference of knots is also presented in Figure 14.

5. Results

Figure 13 shows simultaneously the YZ position of knots where the biggest result difference has appeared. For comparison to the idealized equal prestressing force of 1.0 kN in all elements of minimal net, the forces in the element with biggest result differences were calculated on the basis of presented numerical and experimental methods, and are shown in Table 1. It can be seen that nearest approximation of minimal net was achieved by measuring of model 1, which can be reached only by several iterations of force density method. Experimental results achieved by model 2 are due to the XY plane point definition errors the least precise, but as it is shown in Figure 14, the shape approximation is still satisfying.

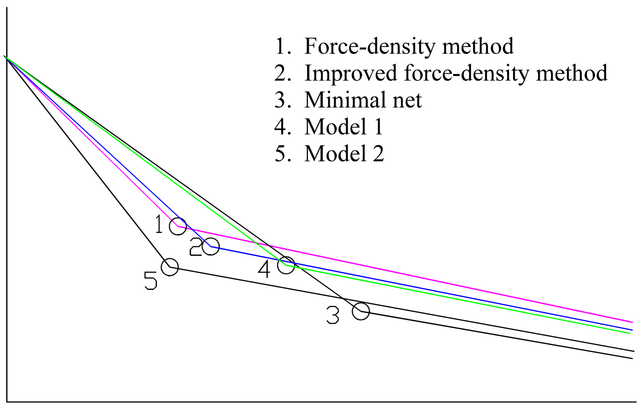

Figure 13. Horizontal position of knot 1.3

.

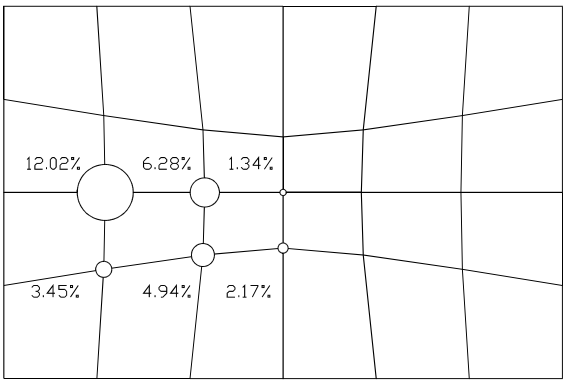

Figure 14. Knot deviations between experimental results and minimal net

Element	force-den.	imp. rez.	Model 1	Model 2	min. net
1,3-2,3	0.7952	0.8683	0.9015	0.7892	1.0001

Table 1. The comparison of prestressing force in element 1.3 – 2.3 calculated on the basis of numerical and experimental results.

6. Conclusion

The results achieved by all experimental and numerical methods are usable approximation for further numerical iterative solving of non linear equations in the form-finding process of the tensile structures. The best agreement between experimental results and the form of minimal net has been performed with the cable-model (model 1). On the other hand, the model 2 is easier to produce, but it is complicated to ensure realistic boundary conditions. Because of the high measurement accuracy and simple handling, used topometric sensor Atos is very suitable for complex 3D shape measurement. The presented procedure shows the possibility of economical integration of architectural models into the form-finding and structure optimization process. The esthetical requirements of a tensile structure can bee satisfy by using of physical model, which shape is then precisely optically measured (digitized) for further numerical analysis.

Literature:

[1] Dvornik, J., Lazarević D., Prednapregnute gipke konstrukcije od užadi i tkanine, Građevinar 47(1995)4, 185-199, Zagreb.

[2] Leonhardt, F., Schlaich, J., Vorgespannte Seilnetzkonstruktionen - Das Olympiadach in München, Der Stahlbau, 9(1972), 257-266.

[3] Linkwitz, K., Combined Use of Computation Techniques and Models for the Process of Formfinding for Prestressed Nets, Grid Shells and Membranes, Internationales Symposium Weitgespannte Flächentragwerke, 3, 84-100, Stuttgart, (1976).

[4] Mollaert, M., The Design of Textile Structures, Euromech 334, 296-310, Lyon, (1995).

[5] Schek, H.-J.,Gründig, L., Steidler F., Matematische Methoden der Netzberechnung und Begründung des Kraftdichtenansatzes, Internationales Symposium Weitgespannte Flächentragwerke, 1, 3.8, Stuttgart, (1976).