Column Strength Curves for the Members of Reticular Domes and its Application to Structural Design

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Abstract

The present investigation proposes a set of column strength curves for the members in the reticulated domes. The column strength curves are prepared for proportioning of the member cross sections in single layer reticular shells considering the elasto-plastic behaviors, the member instability and the global shell buckling with imperfection sensitivity together with the bending rigidity for connection at nodes.

In the procedure, first, (1) a linear elastic analysis is necessary to obtain the stresses in each member under ultimate design loads, second, (2) a second-order elastic analysis is required, but a single time, to estimate the second order effects of deformations on stresses, especially on bending moments and to estimate equivalent buckling slenderness ratio of each member. Third, (3) the column strength curves are applied to estimate the axial strength of each compressive member. The fundamental parameters for the column strength are the equivalent slenderness ratio for column buckling and the knockdown factor for reflecting shell-like buckling. Fourth, (4) the cross section of each member is proportioned based on the stability check by using the column strength and as well as on the strength check by using strength interaction between the axial force and bending moments.

1. Concept of Column Buckling Strength and Knockdown Factor

Shallow reticular domes behave almost similarly to continuum shells and are often subjected to geometrical nonlinearities and imperfection sensitivities before critical states. Their elastic stability has been much investigated based on not only a shell analogy but also a direct nonlinear analysis of reticular domes using FEM. However, there are very few investigations on how the member cross sections of reticular domes should be proportioned against elasto-plastic buckling under design ultimate loads.

1.1 Column buckling in tall buildings

Consideration of elasto-plastic buckling of a mild steel column under compression without eccentricity gives a column strength curve, for examples, the curves illustrated in Fig.1.

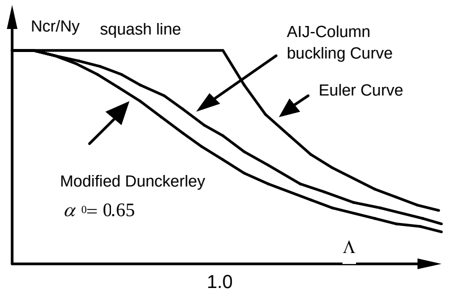


Fig.1 Column strength curves for straight columns

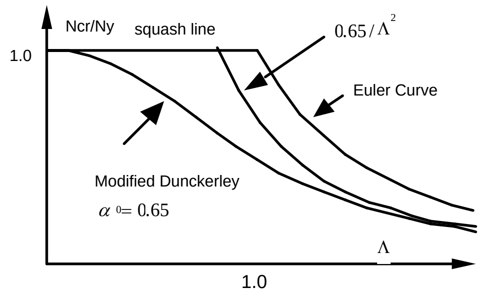


Fig.2 Modified Dunckerley Curve based on IASS recommendation[2]

The curves are expressed in terms of the generalized slenderness, Λ, being a main parameter described as follows.

$$\Lambda = \sqrt{N_y / N_{CR}^{LIN}} \tag{1}$$

where we need the linear buckling axial force N_{CR}^{LIN} and the axial plastic capacity N_y.

In ordinary design of columns, both the quantities, N_{CR}^{LIN} and N_y are calculated based on design recommendations. N_y must be still same in case of reticular shells and then we have a question on how we calculate the linear buckling axial forces for the members in reticulated shells. And again we meet a problem on how we define those column strength curves for reticular shells with shell-like imperfection sensitivity and with semi-bending rigidity at nodal connections.

The shell buckling with imperfection sensitivity is reflected in design curves[1)] for shells under normal pressure by using a knock down factor α_0. Two ways might be possible for expressing the imperfection effects for reticular shells on the basis of α_0. One is a way by which the shell slenderness is expressed as follows.

$$\Lambda_S = \sqrt{N_y / \left(\alpha_0 N_{CR}^{LIN}\right)} \tag{2}$$

And if in this way, a same design curve similar to those for straight columns of buildings might be available.

1.2 Modified Dunckerley formula for shell-like buckling based on a knockdown factor

An alternative way is to base the method on the concept of IASS recommendation on reinforced concrete shells and folded plates[2)] (IASS 1979), that is, the modified Dunckerley method in the recommendations. The equation can be expressed by Eq(3) and can be drawn in Fig.2.

$$\left[\frac{\left(N_{CR}/N_y\right)}{\left(\alpha_0/\Lambda^2\right)}\right]+\left(N_{CR}/N_y\right)^2=1.0 \qquad (3)$$

1.3 Previous studies for knockdown factors of the column strength for the members of reticular shells

Several investigations[3,4,5,6,7,8)] for reticular domes under vertical loading have revealed that the elasto-palstic buckling loads, Pcr per node, can be interpreted as the axial compressive capacity, Ncr/Ny, in terms of generalized slenderness of a particular member which is considered to be most relevant to buckling. The condition that the member is most relevant to buckling is interpreted as the condition that the member is subjected to yielding faster than any others in the dome. That is, the ratio of the linear

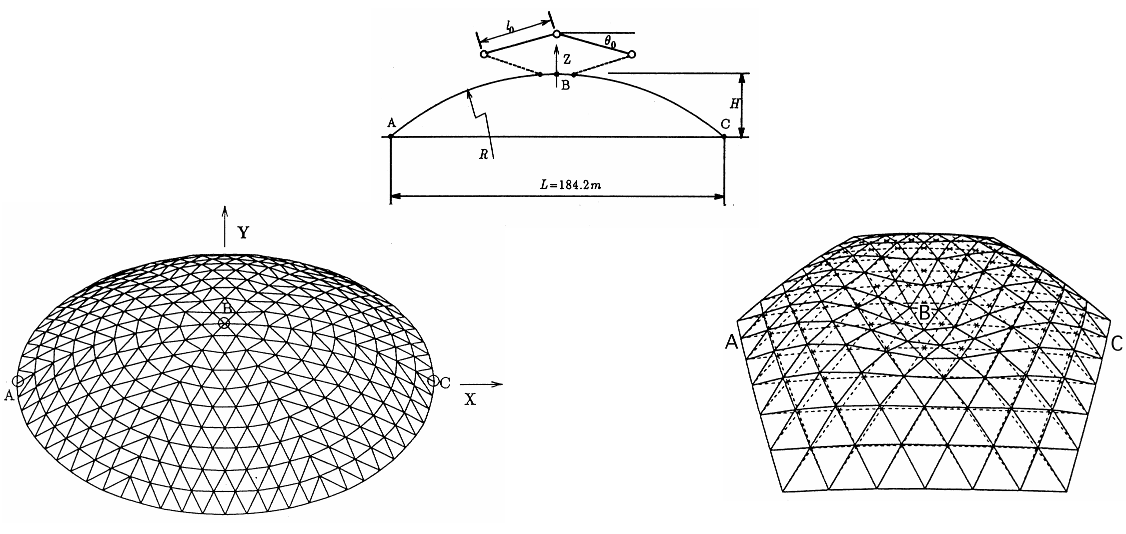


Fig. 3 Geometry of reticular dome roofs; θ_0 is half suspended angle for members on meridians.

buckling stress, N_{CR}^{LIN}, to the axial capacity, Ny, becomes the large than any others. Accordingly, the smallest value Λ among many members is adopted as the representative slenderness as given in Eq.(1).

Fig.4 represents the previous results[4,5,6,7)] obtained for reticular domical roofs on two types of plans, hexagonal and circular as shown in Fig.3, where the connections are assumed completely rigid in some cases and semi-rigid in other cases. The slenderness ratio of members, λ_0, ranges between 30 and 120. The subtended half angle, θ_0, ranges up to 5 degrees.

In Figs. 4.1 and 4.3, rigid-connection at nodes is assumed for end-connections of the members in domes. Geometric imperfections of wi=0.2×te , te being an equivalent shell thickness, are considered in Figs.4.1 and 4.2.

$$t_e = 2\sqrt{3} \times r_g \quad ; \; r_g = \sqrt{I_p / A_p} \tag{4}$$

The subtended half angles, θ_0 , for members on the meridian of ABC in Fig.3 range between 1.0 and 2.5 degrees in the Figs. 4.1 from the results by Kato et al.[6] (Kato et al. 1993) for dome roofs under uniform lateral loading on hexagonal plan, and range between 3.0 and 5.0 degrees in Fig.4.3 from the results by Ogawa et al.[7](Ogawa et al. 1998) also for hexagonal domes under uniform and non-uniform lateral loadings. The figures clearly describe the difference between the results for θ_0 less than 2.5 degrees and those for θ_0 equal or greater than 3.0 degrees. In case of the domes with θ_0 equal or greater than 3.0 degrees, we can see that the elasto-plastic buckling loads are almost determined due to the member collapse, not due to shell buckling, since the calculated column strength curves are almost coincide with the squash line, that is 1.0, or the linear elastic buckling curve, $1/\Lambda^2$. The equivalent buckling length ,Fig.b, based on linear buckling analysis, however not fully shown in the present paper, tends to almost each member length irrespective of θ_0 and λ_0 when θ_0 is equal or greater than 3.0 degrees.

The Fig.4.2 represents the results[5] (Kato et al. 1994) in case of domes with semi-rigid connection at nodes on circular plan, which are assumed to be under uniform lateral loading. In the case of semi-rigidity of connection, the modified generalized slenderness, Λ_{mod}, is adopted described by the following equation.

$$\Lambda_{mod} = \Lambda\sqrt{1/\varepsilon(\kappa)} \quad ; \; \kappa = K_B / \left(EI_P / \ell_0 \right) \tag{5}$$

where the parameters KB, EIp and ℓ_0 represent the bending rigidity for connection, member bending rigidity and member length, and κ represents the rigidity ratio of the bending spring to member bending rigidity.
The quantity $\varepsilon(\kappa)$ stands for the effect[5] (Kato et al. 1994) to show the ratio of buckling load decrease due to semi-rigidity at connection and is approximately represented by the following equation.

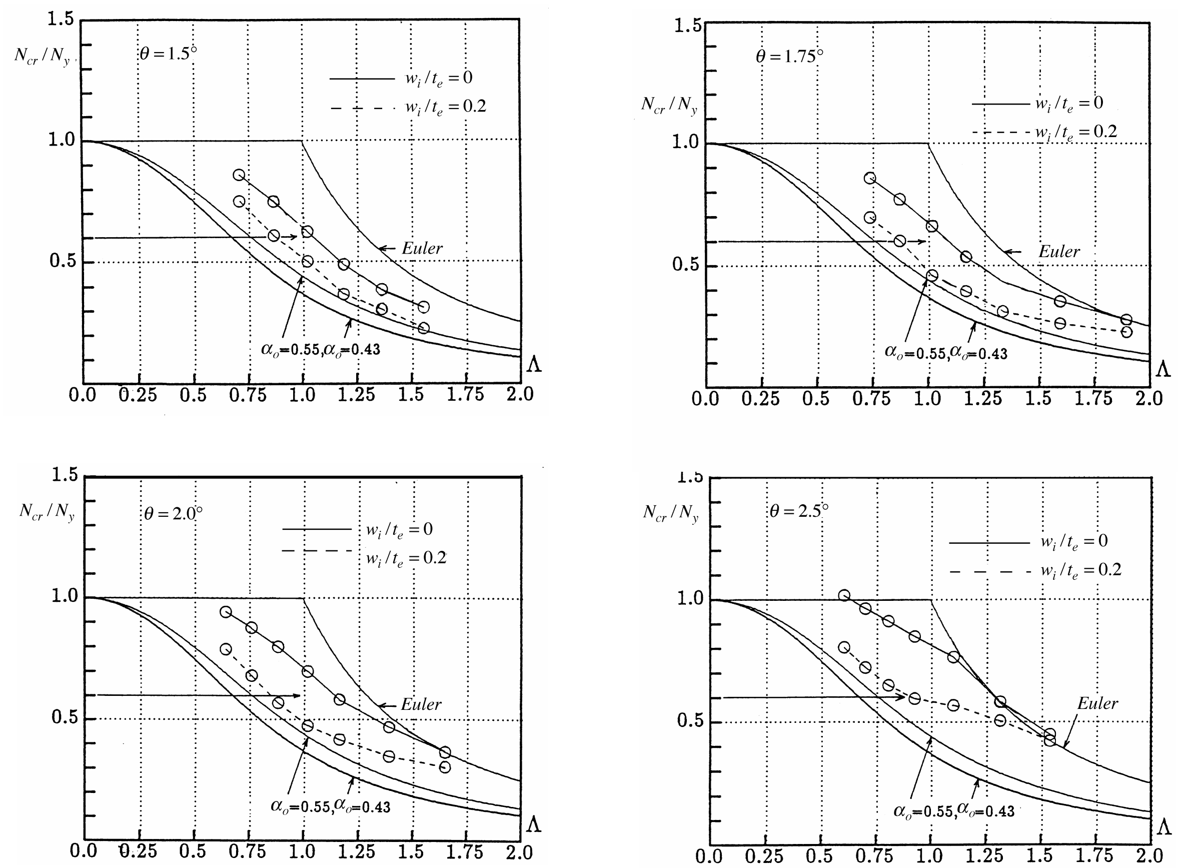
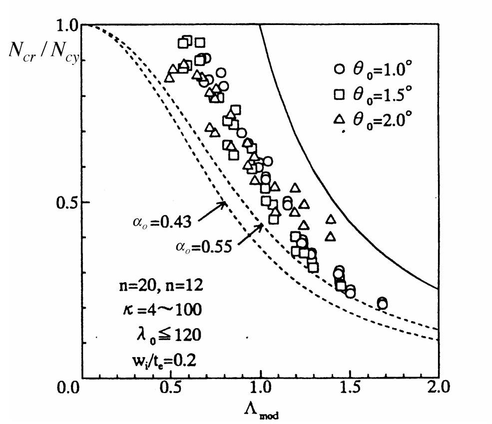


Fig. 4.1 Column strength curves for different half suspended angles θ_0 in case of the domes in Fig.3.1 (Kato 1993, Shibata 1992)



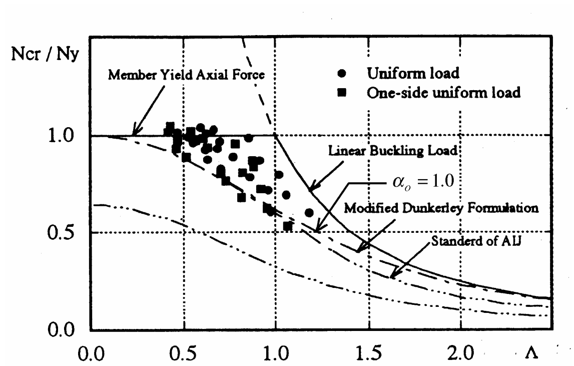


Fig.4.2 Column strength curves for domes composed of members semi-rigidly connected at nodes(Kato 1994, 1995)

Fig.4.3 Column strength curve for domes with θ_0 equal or greater than 3.0 degrees (Ogawa 1998)

$$\varepsilon(\kappa)=\begin{cases}(0.30\log_{10}\kappa+0.30) & for \quad 1\le\kappa\le 10\\ (0.05\log_{10}\kappa+0.55) & for \quad 10\le\kappa\le 100\\ (1.00) & for \quad \kappa\le 100\end{cases} \tag{6}$$

Fig.4.2 proves that the column strength curves are almost same as the ordinary ones for rigid connections when the modified generalised slenderness Λ_{mod} is adopted to express the column strength.

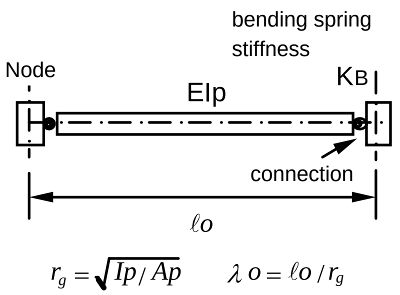


Fig. 5 Modeling for member

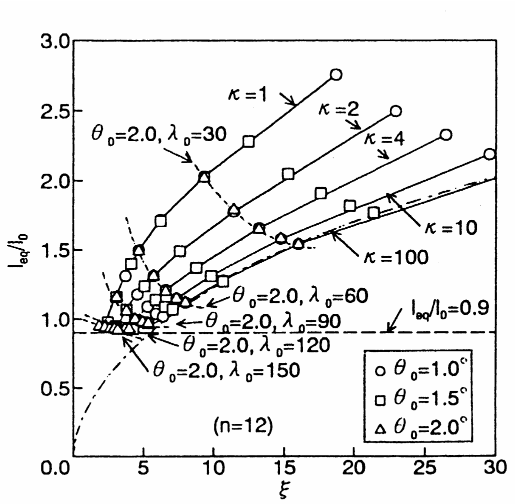


Fig.6 Equivalent buckling length lcr

1.4 Interpretation of equivalent buckling length based on the generalized slenderness

The generalized slenderness can be related to the equivalent buckling half wave length ℓ_{CR}, as similarly to ordinary straight columns under compression.

$$\Lambda = \ell_{\mathrm{CR}} / \ell_{\mathrm{y}} \quad ; \quad \ell_{\mathrm{y}} = \sqrt{\pi^2 \mathrm{EI_p} / \mathrm{N_y}} \tag{7}$$

When lcr is identified for every member in reticular domes, accordingly dimensioning of members in domes might be possible, as similarly to the members in tall buildings, based on the generalized slenderness Λ. The examples for equivalent buckling length calculated by linear buckling analysis are shown in Fig. 6, where a parameter ξ is used for representing shell-likeness.

$$\xi = 12\sqrt{2} / \left[\theta_0\lambda_0\left(1+2/k\right)\right] \tag{8}$$

For the parameter ξ less than 5 under a condition that κ is greater than 4, the equivalent buckling length is almost same as the member length, leading that, in these cases, reticular domes buckle not as shells but buckle like straight columns in tall buildings.

2. Proposal of Column Buckling Strength for Reticular Domes and Proportioning of Members against Buckling

2.1 Column strength curves for reticular domes

According to the discussions on previous researches, the column buckling curves for each member constituting domes may be defined as follows.

$$\left[\left(S_e \times N_{CR} / N_y \right) \Big/ \left(\alpha_0 / \Lambda_{mod}^2 \right) \right] + \left(S_p \times N_{CR} / N_y \right)^2 = 1.0 \qquad \text{(EQ-1)}$$

$$\Lambda_{mod} = \Lambda \sqrt{1/\varepsilon(\kappa)} \;, \quad \kappa = K_B \Big/ \left\{ EI_p / \ell_0 \right\},$$

Table 1 α_0 for reticular domes

	$\theta_0 \leqq 2.5$ deg.	$2.5\text{deg} \leqq \theta_0 \leqq 3.0\text{deg}$.	$\theta_0 \geqq 3.0\text{deg}$.
neglegible geometric imperfection	0.65	interpolation	0.75
intermediate imperfections	interpolation	interpolation	interpolation
geometric imperfection wi=0.2 te	0.55	interpolation	0.65

and the knockdown factor α_0 may be classified as follows approximately illustrated in Table 1. However, we need more discussions on the appropriate values for the knockdown factors and on how to assume the equivalent buckling length for each constitutive member of domes and on the factors of safety.

Second-order elastic analysis for equivalent buckling length: We consider an approximate method for assuming the equivalent buckling length. We will adopt a second-order elastic analysis. The procedure is explained as follows.

(1) Assumption of the ultimate design load per nodes, Pult, based on a nominal dead
load and other loads in issue.
(2) Execution of a first-order elastic analysis to obtain axial forces Nd, bending moments
Mdy and Mdz about y and z axes of each member, under the ultimate design load.

$$[K_L]\{d\} = \{P_{ult}\} \tag{EQ-2}$$

Here [KL] means the linear stiffness matrix of first-order.
(3) Execution of second-order elastic analysis, but a single time, to estimate the $P-\Delta$ effects of nonlinearities on the axial forces and bending moments under Pult.

$$[[K_L]+[K_G(P_{ult})]]\{d\} = \{P_{ult}\} \tag{EQ-3}$$

where $[K_L]+[K_G(P_{ult})]$ is the secant stiffness matrix at the loading level of Pult. From the displacement d in (EQ-3), the axial forces Nd* and mending moments Mdy* and Mdz* are calculated. By adopting the bending moment Mdy* around an strong axis, the approximate linear buckling axial force N_{CR}^{LIN} is calculated as follows[8,9] (Kato et al 1997,1998).

$$N_{CR}^{LIN} = N_d\left\{\left|M_{dy}^* - M_{dy}\right| + \left|M_{dy}\right|\right\}/\left|M_{dy}^* - M_{dy}\right| \tag{EQ-4}$$

If N_{CR}^{LIN} is greater than the Euler buckling load Ne for a member with a buckling length equal lo its member length, N_{CR}^{LIN} is replaced by Ne. Then the generalized slenderness Λ_{mod} is calculated, given below, for each member.

$$\Lambda_{mod} = \Lambda\sqrt{1/\varepsilon(\kappa)}\ ;\ \Lambda = \sqrt{N_y/N_{CR}^{LIN}}\ ;\ \kappa = K_B/\left\{EI_p/\ell_0\right\} \tag{EQ-5}$$

where Eq.6 is used to evaluate the reduction of buckling loads due to semi-rigidity of connection.

Dimensioning of members:

The cross section of each member is dimensioned by the following two equations; one is the strength check and the other is the stability check.

$$\text{Strength check}\qquad \left(\frac{N_d}{N_y}\right)^2+\left[\left(\frac{M_{dy}{}^*}{M_{py}}\right)^2+\left(\frac{M_{dz}{}^*}{M_{pz}}\right)^2\right]^{1/2}=1.0$$

$$\text{Stability check}\qquad \frac{N_d}{N_{CR}^{LIN}}\leq 1.0 \qquad\qquad \text{(EQ-6)}$$

where M_{py} and M_{pz} are the plastic moments for the y and z axes.

For the dimensioning another judgements will be required especially for members with so small axial forces and bending moments. In the present procedure, following judgements are provided to ensure smooth continuity of member rigidity.

$$\begin{array}{lcl} N_{d(min)}=\gamma\times N_{d(max)} & ; & N_{d(min)}=\gamma\times N_{d(max)} \\ M^*_{dy(min)}=\gamma\times M^*_{dy(max)} & ; & M^*_{dy(max)}=\mathrm{Max}\left(\left|M^*_{dy}\right|\right) \\ M^*_{dz(max)}=\mathrm{Max}\left(\left|M^*_{dz}\right|\right) & ; & M^*_{dz(max)}=\mathrm{Max}\left(\left|M^*_{dz}\right|\right) \end{array} \qquad \text{(EQ-7)}$$

Here some consideration is needed to define the magnitude of γ, a value around 0.5 being recommended in the present paper based on the previous studies[8.9] (Kato et al. 1997, 1998).

The factors for safety:
On what values for examples Se and Sp in IASS recommendation should be recommended in actual design as the factors of safety, is an actual issue to be discussed, and an alternation is to adopt the values recommended in some appropriate ones such as IASS recommendation.

3. Conclusions on Feasibility of the Proposed Procedure for Reticular Domes

The discussions covering the definition of column strength together with the assumptions for both the knockdown factors and slenderness ratios prove the present proposal to be effective for designing reticular domes under ultimate design loads. Several examples[8,9] have revealed that the reticular domes designed by a similar procedure to the present one, but with a little different knockdown factors, satisfy, to a great extent, the required ultimate design loads.

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