# Cutting Pattern Generation of Structural Membranes

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**Abstract**

This paper sets out a simple design procedure for the determination of tension membrane shapes while at the same time generating the cutting patterns of the fabric of the surface. It does this by pre-defining the requirements of the cutting pattern which is set out by the designer on an initially flat surface. A cable net is used for the approximating surface and the cables approximate the warp and weft directions of the fabric. A companion, triangular mesh, is also generated to be used for calculating nodal forces due to surface pressure and dead loads, and also for use in the visualization of the membrane shape. The present work sets out the procedure and gives examples of results of the formfinding of a surface with a number of different cutting patterns.

## 1. Introduction

The design of membrane structures involves three phases. The first is the determination of the initial equilibrium geometry for the membrane shape, and has been termed `formfinding'. Once a satisfactory shape has been obtained, the cutting patterning is carried out. In this the membrane surface is divided into strips to be cut from the rolls of the fabric material and then fabricated to give the final shape. The third phase involves the computation of the stresses in, and deformation of, the completed membrane under its various load conditions.

This paper focuses on the aspects of both formfinding and the cutting pattern for the pre-stressed membrane structure. The authors' main concerns are the requirements for obtaining an optimal cutting pattern for an efficient use of the structural properties of the membrane material avoiding wrinkling and at the same time having minimal waste of material and minimal length of seams. A computer method is outlined and an example applying the method where these aspects are addressed to generate an overall acceptable and simply derived ``optimal'' cutting pattern.

The practicalities and economics of fabrication dictate that the patterns produced should make full use of the roll width of the material [1]. To generate details for the fabrication of the structure, the structure surface has to be formed by joining shaped strips of fabric that are less than the roll width of the membrane fabric as supplied by the manufacturer. On the other hand, cost considerations require efficiency in the use of material. Hence, strips should be chosen so that as near as practicable the full width of the roll is always utilized. This in turn reduces the number of strips required and also the total length of seams to be joined. For surfaces with pronounced curvature, the developed strips may display curved edges, so creating some waste of material while fitting within the width of the roll of fabric. With this in mind Ishii [4] proposed cutting along geodesic lines on the surface as a means of producing manageable strip profiles. Various computer techniques have been researched and developed to assist engineers on the methods of locating geodesics and patterning [6] [10] [3] [1] [8] [5] [9] [2]. The motivation behind this strategy is readily appreciated from the study of geodesics.

## 2. Geodesic lines

In the design of a tension structure, geodesic lines are often used as cutting lines of the membrane fabric. For the reasons of material economy and accuracy, the seams of panels should ideally follow geodesic paths over the curved stressed surface. These geodesic paths are trajectories which a flat tape of material will follow without shearing. Herein a simple numerical method based on physical intuition has been developed to produce geodesic lines on the fabric surface. The basic concept is that in the original flat position the designer starts with a cable mesh for which one set of cables coincides with the geodesic lines which will be used for the cutting pattern of the final membrane. The technique for maintaining these geodesics on the deflected surface is as follows:

1. In the mesh generation program identify the side of a patch, from which the geodesic tree can be established. Store the topology of this geodesic tree.

2. In the mesh generation program set the widths of strips to be equal to the width required by the designer. To be less than the cloth width in the final fabrication, an allowance is made for some increase in dimensions as the shape is formed. For example, if the widths of strips in one patch are *a, b, c*..., usually these widths from the initial mesh generation are not equal, i.e., $a \neq b \neq c \neq \cdots$. Suppose that the required width is d, and set $a = b = c = \cdots = d$, such that

$$a\,(1+\frac{b}{a}+\frac{c}{a}+\cdots)=nd \tag{1}$$

   in which *n* is the number of strips of one patch.

3. In the flat position, each straight cable line, being the shortest distance between its end points, is a geodesic line.

4. Having started from the initial geodesic position small increments in transverse displacements will not radically alter the positions of the geodesics, which can be easily re-established by the procedure given in step 5.

5. In the formfinding process, after a number of increments have been completed, calculate the length of the original geodesic and also of two adjacent lines on the surface, to left and right of this position, and displaced by a small amount from this line. A parabolic shape is assumed for each of these lines with respect to the displaced original geodesic. The lengths of these three lines are compared, and used to locate the best approximation to the correct geodesic. Thus, if the three calculated lengths are $l_1, l_2, l_3$ respectively, with $l_2$ being the initial line and $l_1, l_3$ the lengths of the lines displaced by the amount $\pm 1$, in $\xi$ co-ordinates, then, for any intermediate $\xi$ value, the interpolated length is given,

$$l=\frac{1}{2}\xi(\xi-1)l_1+(1-\xi^2)l_2+\frac{1}{2}\xi(\xi+1)l_3 \tag{2}$$

6. If the geodesic line lies in the range $\xi = \pm 1$, then the minimum length is given at

$$\frac{dl}{d\xi} = 0 \tag{3}$$

From this equation the nodal co-ordinates of the next approximation to the geodesic line can be calculated. If either $l_1$ or $l_3$ is shorter than $l_2$, then the offset distances must be increased slightly, so that the geodesic line is included in the new range of $l_1 - l_3$ values.

7. Shift the strip lines to the geodesic line positions. The new co-ordinates $X_i$ can be obtained by a simple linear shift,

$$X_i = X_{iI} + (X_{iJ} - X_{iI}) \frac{\Delta}{L_{IJ}} \tag{4}$$

where *I*, *J* are adjacent lines of the strip, $\Delta$ is the distance the node must be shifted and $L_{IJ}$ is the length of the element across the strip. This will disturb the equilibrium of the system. This is compensated for by the procedures 8 and 9.

8. Include the required prestress *F* in each element by changing the original length of elements as follows

$$L_{oi} = \frac{L_i}{1 + F/EA} \tag{5}$$

where $L_i$ is current length of the element, *E* is the Young's modulus and *A* is the section area of the cable elements.

9. It usually requires only one iteration to reach a new equilibrium position. Then the new geodesic pattern has been established in the formfinding process.

10. Check the width of each strip to see if the maximum strip widths are still equal. Usually they are no longer equal after formfinding and geodesic determination. If they are not equal, then the initial widths of the strips are reset. For example, after formfinding the widths of the strips $a_1, b_1, c_1, \cdots,$ then

$$\frac{b}{a} = \frac{a_1}{b_1}, \qquad \frac{\mathrm{c}}{\mathrm{a}} = \frac{a_1}{c_1}, \qquad \cdots \tag{6}$$

according to Equation (1),

$$a\,(1 + \frac{a_1}{b_1} + \frac{a_1}{c_1} + \cdots) = nd \tag{7}$$

The initial widths of the strips are modified and the initial mesh co-ordinates are adjusted accordingly.

11. Repeat the formfinding and geodesic determination. Usually after one repetition a satisfactory result is obtained. The incremental displacements are applied in this way until the final total displacements are reached. The refined positions of the nodes in this final position are then used as the basis for the calculation of the cutting patterns.

The fabrication geometry is obtained from final formfinding analysis that includes the detailed geometry of all system points and the allowance for the initial strains in the fabric material.

## 3. Cutting pattern generation

After the edges of a strip have been identified by determination of the geodesics, the flattening of the strip can then proceed. The quality of the cutting pattern is of decisive importance for the usefulness, economy and appearance of a stressed membrane structure.

Because the strip is defined along its edges only by the geodesic nodes, the strategy that is adopted is essentially based on the triangulation of the strip using these edge nodes. A sequence of triangular elements associated with a strip is unfolded from three dimensions into two dimensions to obtain the cutting pattern for fabrication. Sufficient points must be chosen in each side so that the shape of the strip is not unduly distorted. The selection of points to form each successive triangle is effected so that the next point to be chosen is from the side in which the percentage of unused points is the greater. The procedure of flattening of fabric strips is as follows:

- In order to develop the three dimensional(3D) curved strip on to a plane each side of the strip is divided into a pre-assigned number of divisions. With these divisions the strip is then divided into fine triangles with 3D coordinates but whose planar dimensions are known. Then these fine triangles are mapped sequentially onto the plane. For each strip the first point $(a_1, b_1)$ of the first triangle in the development will be from the end of the side with longer length. The second point $(a_2, b_2)$ of the first triangle is from the end of the side with shorter length which shares the same cable element with the first point in the three dimensional surface. It is easy to locate their two dimensional(2D) co-ordinates according to their 3D relationship. The 2D co-ordinates *x*, *y* of the third point of the first triangle are obtained from

$$(x - a_i)^2 + (y - b_i)^2 = s_i^2 \quad i = 1, 2 \tag{8}$$

  in which $s_i$ is the distance between the third point and each of these two points.
- To form the next triangle adjacent to the previously developed triangle, the process is repeated, taking the first side from that already calculated, so the two vertexes of the new triangle are two corner nodes of the previous one and the third vertex of the next triangle is chosen from the side of the strip with fewer segments.
- This process is repeated until all points from both edges of the strip have been selected and thus the strip has been flattened onto the plane.
- After the strip has been developed onto the plane, it may be laid out on the material roll width, so as to conform with the strip width of the material. Coordinates are calculated to allow easy setting out on the roll of the material.

Because this procedure is a linearization of a non-linear process the subdivision must be fine along a geodesic. For example one hundred points can be used to produce accurate flattening of the strip.

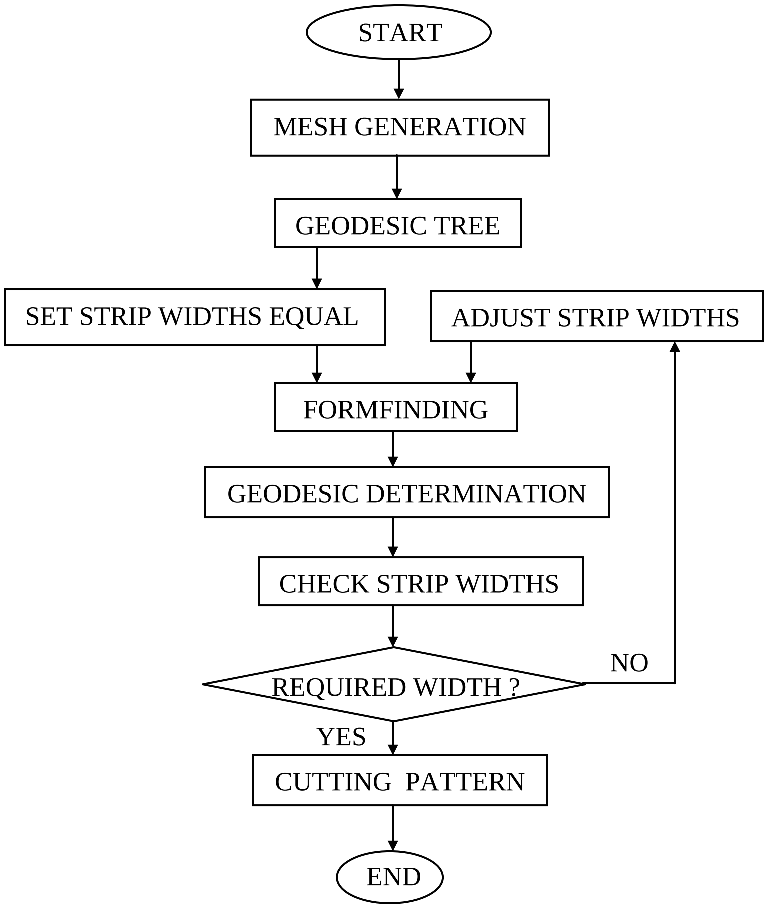



Figure 1 Cutting pattern flow chart

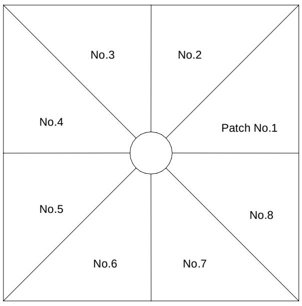
## 4. Compensation factor

Because the unfolded patterns obtained from the surface development still represent the final stressed state of the membrane, suitable compensation factors must be used to allow for the stretching of material from the unstressed state to the stressed and installed condition to produce sufficient tensions in the membrane to give it adequate stiffness. After all the points for a strip have been developed onto the plane, the appropriate compensation factors can then be applied in the longitudinal and transverse directions corresponding to the warp and weft weave of the fabric. The compensation factors are considered as follows,

$$x' = x - \alpha_x x \qquad y' = y - \alpha_y y \tag{9}$$

in which $x$, $y$ and $x'$, $y'$ are node co-ordinates of a strip before and after considering the compensation factor respectively while $\alpha_x$, $\alpha_y$ are the reduction ratios along weft and warp directions. Values of these factors depend on the material being used and may be in the region of 3 percent. They are used to produce permanent strain in the fabric for the load analysis stage. Finally the dimensions of each strip are presented in a manner to allow easy setting out on the fabric (see Figures 12, 13).

## 5. Two numerical examples



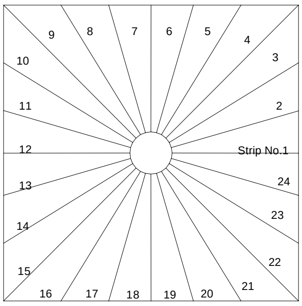
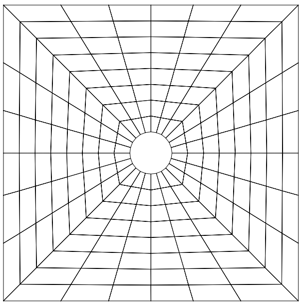




Figure 2 Mesh generation with patches

Figure 3 Mesh generation with strips



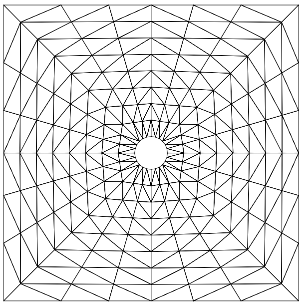
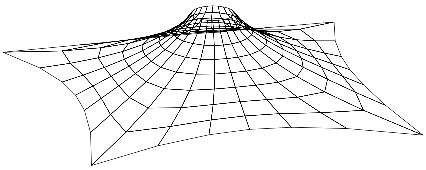


Figure 4 Mesh with cable elements

Figure 5 Mesh with triangular elements



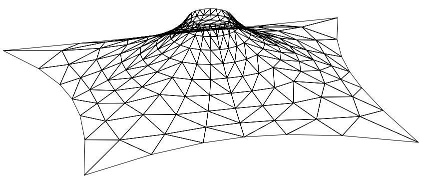


Figure 6 Formfinding with cable elements

Figure 7 Shape presented with triangular element

Figure 8 Formfinding showing fabric material

Figure 9 Split strips and Formfinding

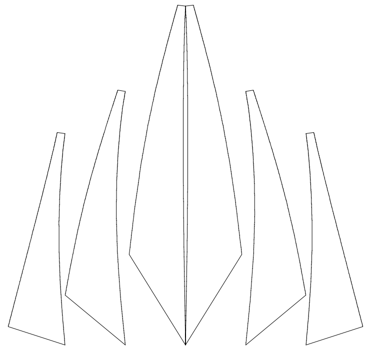


Figure 10 Flattened strips of patch No.1 & No. 2

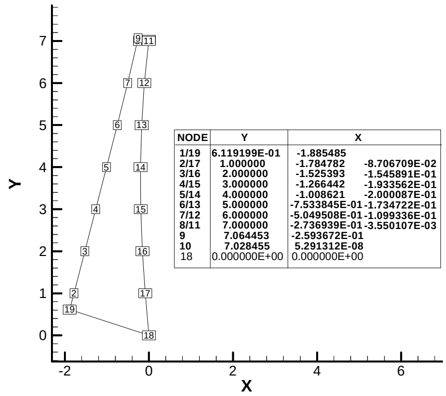



Figure 11 Flattened strip No. 1

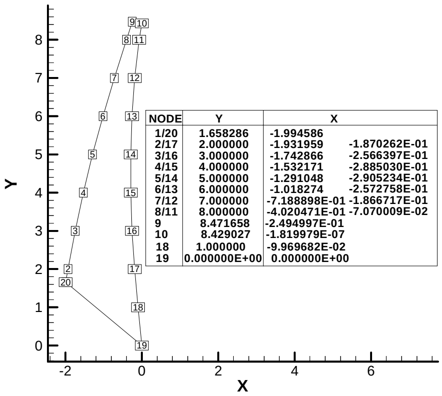



Figure 12 Flattened strip No. 2

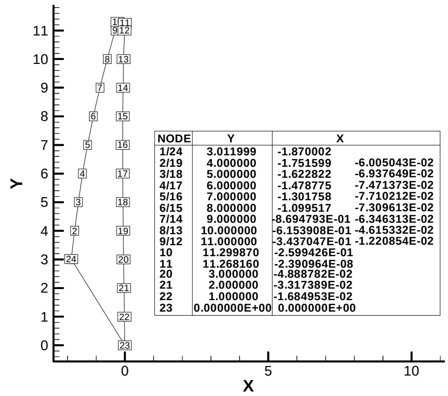



Figure 13 Flattened strip No. 3

Figures 2-5, show the design procedure starting with a plan view of the area to be covered. Mesh generation by patches (Figure 2) can be obtained from the input of the co-ordinates for the corner nodes for each patch. Then by interpolation, mesh generation by strips (Figure 3) and mesh generation by cables (Figure 4) are obtained. Before formfinding starts the maximum widths of all strips are set equal in order to assure the final widths of strips will be approximately equal. It should be noted that instead of the triangle elements as shown in Figure 5, only two node cable elements in Figure 4 [7] are used to generate the mesh. With the cable model however, a companion triangular surface model Figure 5 is generated in order to calculate the node loads on the cable for distributed loads and also to be used in representing the geometric continuity of the membrane surface.

Figure 6 shows the formfinding of a conical-shaped membrane structure. A finite element mesh, of the two-node elements $14\times14$(m) square and with a centre circular opening of 2(m) diameter is first established within the specified boundary of the structure projected on to the horizontal plane. From this flat position, the nodal points corresponding to support, tie-down and pick-up points are given specified displacements in small increments until their target positions are reached. Here 4(m) is used for the final formfinding. More details on formfinding are described by Meek and Xia [7].

The Figure 8 presents the final membrane shape of the real material surface shown using the 3Dstudio software. Strips have their edges formed by geodesics given previously. Furthermore the real design can be viewed when it is necessary to examine the aesthetics of the membrane form. Figure 9 shows the strips of the conical-shaped membrane in their three-dimensional state. For each strip the two side edge lines are located with geodesics. For any construction for membrane structures, no matter of what shape, they are always formed by such strips.

In Figure 10, the flattened strips, using the method described previously are shown, including the compensation factors. Figures 11-13 show detailed data for the cutting pattern for each strip. As examples three sets of strip data for the corresponding patches are shown together with their relevant plots. The required width is 2.1(m). It will be seen that co-ordinates are set out for ease of drafting of the strip on to the roll of fabric.

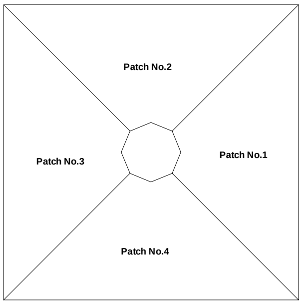



Figure 14  Mesh generation with patches

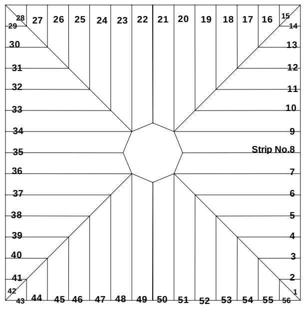



Figure 15  Mesh generation with strips

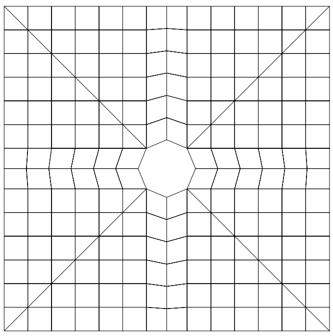


Figure 16  Mesh with cable elements

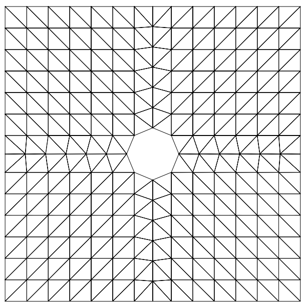


Figure 17  Mesh with triangular elements

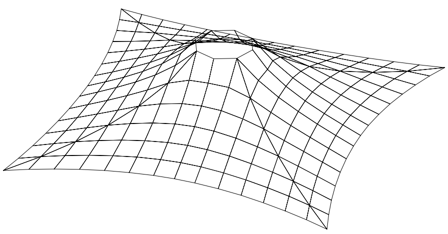


Figure 18  Formfinding

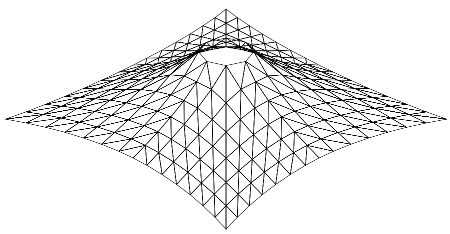


Figure 19  Form represented with triangular surface

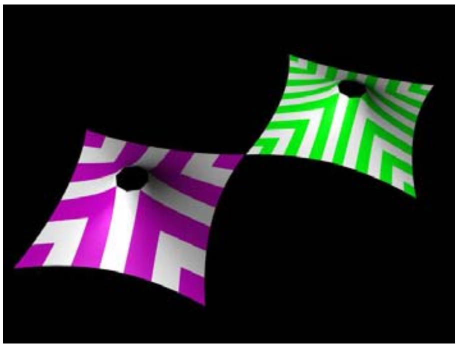


Figure 20  Formfinding with strips distribution

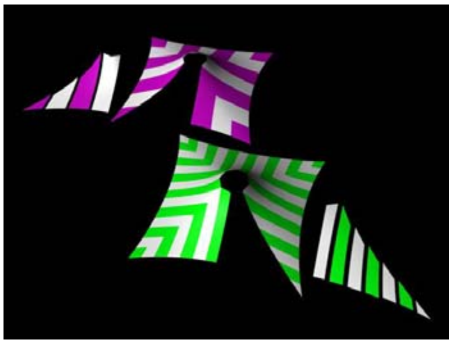


Figure 21  Formfinding with split strips

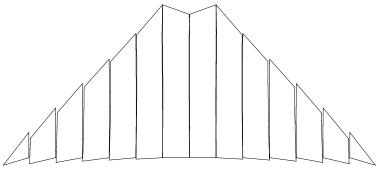


Figure 22  Flattened strips of patch No. 1

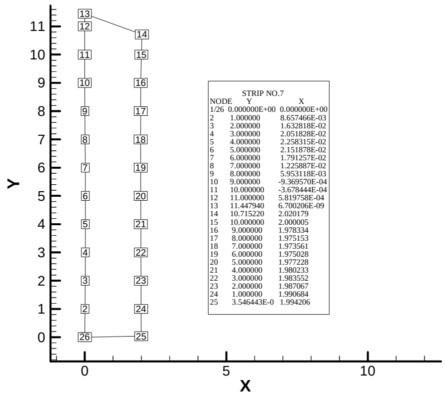



Figure 23  Flattened strip No. 7

Figure 14-15 shows a 28×28(m) square mesh with four patches in which the strips are essentially parallel to the edges rather than radial direction. This makes the formfinding somewhat different from that of Example 1.

Figures 16-19 show the mesh generation and the formfinding of this type of membrane. From the flat position shown in Figure 16, with four corner nodes fixed, the six central nodes are given specified displacements in small increments until their target positions(here 4m) are reached. Geodesics are traced during the formfinding procedure. It is noticed that the length of the edge line of some strips is slightly shorter than before geodesics were located. This can be explained by the fact that the mesh with this type of strip distribution is naturally along geodesics. With the cable mode a companion triangular surface model (Figure 17 and Figure 19) is generated in order to calculate the node loads on the cable for distributed loads and also to be used in representing the geometric continuity of the membrane surface. The width of the strips depends on the material width from manufacturers.

Figures 20-21 show formfinding with strip distribution and formfinding with split strips.

Figure 22 shows flattened strips of patch No.1 and Figure 23 shows flattened layout for the strip No. 1 in patch No.1 after the compensation factor has been applied.

It has been calculated that there is a 32% saving of material by using the example 2 model rather than the example 1 model. It can be seen that the fabric structure with this type of strip distribution is more convenient for the cutting pattern and a more economical way of making use of the material because of basically straight strip lines.

## 6. Conclusions

This paper has been devoted to the processes involved in membrane formfinding and the associated problem of producing material strips for the fabrication of tensile membrane structures. The requirements of both economy and accurate surface development, and how these can be best achieved are discussed.

Geodesics were defined and from this definition, intuitive reasoning justified their use in cutting pattern design. A simple method of geodesic cutting patterning, based on a numerical method of locating geodesics, was described [7]. This method provides the designer with flexibility in the design of cutting patterns for membrane structures in that layouts can be predetermined for best visual effect. Proof was given of the validity of using stretch-independent uniform tension cable elements to obtain the definition of geodesics on a predefined surface.

A triangulation of the edge nodes developing the strip onto the plane has been adopted, and is accurate provided that the subdivision is fine. That is, the relative spatial position of a large number of points on the idealized surface is duplicated accurately in the plane. The compensation factor has been considered in the final stage of cutting pattern.

## Reference


[1] C.F.Ong, D.S.Wakefield and M.R.Barnes. Interactive Graphic CAD for Tension Structures. *Computers & Structures*, 41(6):1305-1312,1991.

[2] Wakefield David. Tensyl: the Development of an Integrated CAD System for Stressed Membrane Structures. *International Conference on Lightweight Structures in Architecture, 1st. 1986. Proceedings*, 271-278, Sydney, Australia, August 1986.

[3] Ulrich. Cutting Pattern for Structural Membranes. *International Conference on Lightweight Structures in Architecture, 1st. 1986. Proceedings*, 279-286, Sydney, Australia, August 1986.

[4] Ishii, K. On Developing of Curved Surfaces of Pneumatic Structures. *International Symposium on Pneumatic Structures*, IASS, Delft, 1972.

[5] Peter Kneen. Fabric Structures. *Modern Structural Engineering*, Kneen/1-23, Sydney, August 1984.

[6] J. L. Meek and K. Y. Tan. Post-Formfinding Determination of Geodesic Lines in Cutting Pattern Design for Membrane Structures. *Space Structures*, (2):231-239, 1986/87.

[7] J.L. Meek and Xiaoyan Xia. Shape Finding Analysis of Form Structures. *Application of Numerical Methods in Engineering*, 357-366, University Pia, June 1997.

[8] E. Moncrieff and B. H. V. Topping. Computer Methods for the Generation of Membrane Cutting Patterns. *Computers & Structures*,37(4):441-450, 1990.

[9] Robert Roche, Detler Sulzer and Harald Muehlberger. World Expo 88 - Tension Membrane Structures. *International Conference on Lightweight Structures in Architecture, 1st. 1986. Proceedings*, 940-955, Sydney, Australia, August 1986.

[10] Tan, K. Y. The Computer Design of Tensile Membrane Structures. PhD thesis, *The University of Queensland*, 1989.