GENERATING SHELL SHAPES BY COMPUTER MODELLING

John L. Meek and Xiaoyan Xia
Department of Civil Engineering, The University of Queensland, St. Lucia Qld 4072, Australia
J. L. Meek Tel: 617 33788423 Fax: 617 33654161(w) or 617 33788568(h) e2meek@brolga.cc.uq.oz.au
Xiaoyan Xia Tel: 617 33653508 Fax: 617 33654599 x.xia@mailbox.uq.edu.au

Abstract

The purpose of this paper is to present a method by which compression shell shapes can be obtained by inverting the tension shape obtained by loading a cable net system. A companion, triangulated mesh is also produced simply to obtain nodal forces from distributed loads, and also for the computer visualization of the surface. This work parallels the physical models used by H. Isler for his work on thin concrete shells. The paper gives examples of the mesh shapes generated and the computer visualization of several examples. It demonstrates that the computer modelling is a suitable alternative to the physical modelling process.

1. Introduction

A shell is a double-curved rigid thin skin structure. Unlike conventional, beam and column building design in which the architect defines the structure's shape and the engineer performs the static analysis, in shell design, both aspects are united. This dual task of the design of form and structure is a great advantage for aesthetics and strength and a greater challenge to the designer.

Defining the form is the important first step in shell design. The quality of this choice finally decides the quality of the building. A perfectly shaped shell is efficient since it carries its loads by nearly pure membrane action, whereas a badly shaped shell is hardly able to be saved. There are at least two ways of formfinding in free form shell design. These are physical experiments and computer modelling. For the former, Heinz Isler [1]-[8] has done a great deal of work while the latter approach has not yet been widely used. Therefore it is of significance to be able to form shell structures by computer modelling and to show the possibility that the structures can be constructed in practice.

In a forty year period, Heinz Isler and his colleagues have designed and built at least one thousand concrete shell structures. The shell shapes are not based on geometric concepts, but result from formfinding experiments. The shapes are created automatically by natural laws.

The physical model method by Isler [1] encourages us to look for numerical models in the design of arbitrary shapes for compression shell structures. It has been seen that the possibility of developing this field is of no doubt. In the present paper computer software is developed to simulate these direct experimental processes to show that it is helpful in the creative process of shell design.

Firstly it requires an understanding of the shapes Isler has obtained. The shape performance can be checked by a geometric non-linear analysis. The shell shapes generated herein are based on computer studies. The shapes are created by applying the dominating surface loads on a flat cable net acting as a tension structure and then inverting for the shell stresses to be compressive rather than tensile. Subsequent analysis of the shell for its service loads will estimate maximum stresses from both membrane and bending effects. From the formfinding it has been possible to produce a range of shapes similar to those developed by Isler.

2. Formfinding

A finite element mesh of two-node elements is first established within the specified boundary of the structure projected onto the horizontal plane. From this flat position, with an initial pre-stress the mesh is loaded with a uniformly distributed load or internal inflation pressure in the reverse direction to the real loads applied to the shell and a geometric non-linear analysis is performed. In this way a tension stress membrane is formed. When the loads are reversed, a compression membrane is obtained.

The linear two node cable element is the simplest finite element encountered in structural analysis. For this element the only pure deformation mode is the elongation of the element. Corresponding to this natural deformation mode, a natural force can also be defined. For large displacement analysis the tangent stiffness $[K_T]$ of the element is comprised of two components. The elastic stiffness $[K_E]$ and the geometric stiffness $[K_G]$, which are derived separately in the following, for more detailed description see [9].

Elastic stiffness

The tangent stiffness, $[K_T]$ is calculated from the current deflected position, using the updated co-ordinates as the reference configuration. Member forces are calculated as the sum of the incremental contributions. The theory for the analysis of cable structures in this method is based on small deflections from the current position. The vector co-ordinates of the member ends I, J are given by

$$x_I^T = \{x_{I_1}, x_{I_2}, x_{I_3}\}; \qquad x_J^T = \{x_{J_1}, x_{J_2}, x_{J_3}\} \tag{1}$$

The member direction cosines (current position) are

$$\{c\}^T = \frac{1}{l}\{(x_{J1} - x_{I1})\,(x_{J2} - x_{I2})\,(x_{J3} - x_{I3})\} \qquad i = 1, 2, 3 \tag{2}$$

Where l is the member length. The vector of member displacements is written

$$\{r\} = \begin{Bmatrix} r_I \\ r_J \end{Bmatrix} \tag{3}$$

where r_I and r_J are defined in the same co-ordinate system as the co-ordinate vectors in Equation (1). Hence, if $\{\Delta r_i\}$ is the increment in $\{r\}$ at the i iteration, then the updated nodal co-ordinates of the member are

$$\{x_{i+1}\} = \{x_i\} + \{\Delta r_i\} \tag{4}$$

Similarly, the force vectors at the member ends are given by

$$\{P_E\} = \begin{Bmatrix} P_I \\ P_J \end{Bmatrix} = \begin{Bmatrix} -\{c\} \\ \{c\} \end{Bmatrix} P_N = \{a_N\}^T P_N \tag{5}$$

Where P_N is the member force. Thence,

$$\{\Delta P_{Ei}\} = [a_{N_i}]^T \Delta P_{N_i} = [a_{N_i}]^T k_N [a_{N_i}]\{\Delta r_i\} = [K_E]\{\Delta r_i\} \tag{6}$$

Where $k_N = \dfrac{EA}{l_o}$, is the member stiffness and

$$[K_E] = \frac{EA}{l_o}\begin{bmatrix} \{c\}\{c\}^T & -\{c\}\{c\}^T \\ -\{c\}\{c\}^T & \{c\}\{c\}^T \end{bmatrix} \tag{7}$$

Geometric stiffness

The member displacement vectors r_I, r_J are resolved into components parallel $\{r_{\alpha\|}\}$ and perpendicular $\{r_{\alpha\perp}\}$ to the member. Let $\alpha = I$ or J, so that

$$\{r_\alpha\} = \{r_{\alpha\|}\} + \{r_{\alpha\perp}\} \tag{8}$$

where

$$\{r_{\alpha\|}\} = \{c\}\{c\}^T\{r_\alpha\} \tag{9}$$

thus

$$\{r_{\alpha\perp}\} = \{r_\alpha\} - \{c\}\{c\}^T\{r_\alpha\} \tag{10}$$

It is seen that a measure of the rotation of the member is given by $\{\bar{r}\}$, such that

$$\begin{aligned}\{\bar{r}\} &= \frac{1}{l}\left[\{r_{J\perp}\} - \{r_{I\perp}\}\right] \\ &= \frac{1}{l}\left\{-[I_3 - \{c\}\{c\}^T][I_3 - \{c\}\{c\}^T]\right\}\begin{Bmatrix} r_I \\ r_J \end{Bmatrix}\end{aligned} \tag{11}$$

where I_3 is 3×3 unit matrix. The vectors $\{\Delta P_{IG}\}$, $\{\Delta P_{JG}\}$ are the changes in the global components due to the rotation. Then

$$\begin{Bmatrix} \Delta P_{IGi} \\ \Delta P_{JGi} \end{Bmatrix} = \frac{P_N}{l}\begin{bmatrix} \left(I_3 - \{c\}\{c\}^T\right) & -\left(I_3 - \{c\}\{c\}^T\right) \\ -\left(I_3 - \{c\}\{c\}^T\right) & \left(I_3 - \{c\}\{c\}^T\right) \end{bmatrix}\begin{Bmatrix} \Delta r_{Ii} \\ \Delta r_{Ji} \end{Bmatrix} \tag{12}$$

This is the equation for the ith iteration,

$$\{\Delta P_{Gi}\} = [K_G]\{\Delta r_i\} \tag{13}$$

Combining Equations (6) and (13),

$$\{\Delta P_i\} = \{\Delta P_{Ei}\} + \{\Delta P_{Gi}\} = [K_E + K_G]\{\Delta r_i\} = [K_T]\{\Delta r_i\} \tag{14}$$

Finally, $[K_T]$ is written as the 6×6 partitioned matrix:

$$[K_T] = \begin{bmatrix} [k] & -[k] \\ -[k] & [k] \end{bmatrix} \tag{15}$$

Where,

$$[k] = \left[\frac{EA}{l_o} - \frac{P_N}{l}\right]\{c\}\{c\}^T + \frac{P_N}{l} I_3 \tag{16}$$

Newton-Rhapson type iteration is carried out for the iterative calculations and the displacement criteria is adopted in the formfinding procedure. In this process, since the deflection increments are always calculated from the current position, it is important that a starting position near the next calculation is used, so that $[K_T]$ is a good estimate of the tangent stiffness. In the form finding process it is convenient to chose an initially plane configuration as the initial position.

The geometry obtained is examined to see if the required height for the structure has been reached and if the required shape has been formed. Otherwise it is necessary to adjust some of the input data parameters. From experience usually after a few iterations satisfying results can be obtained. With the cable model a companion triangular surface model is also produced in order to calculate the node loads on the cable system for distributed loads and also to be able to represent the geometric continuity of the membrane surface for graphic presentation.

The blending of the specified pre-stress with the elastic stresses from the imposed displacements will then determine the resulting curvatures of the surface. Using a high elastic modulus, the elastic stresses will dominate and may completely overshadow the effect of the specified pre-stress. Lower elastic modulus will result in increasingly more pronounced radial curvature.

By using the orthotropic material property in the formfinding analysis further fine adjustment to the curvatures of the surface is possible. By making the traverse or hoop stiffness greater than that of the longitudinal or radial, it is possible to induce more stresses in the traverse or hoop direction and this results in an increase in longitudinal or radial curvature.

3. Some computer models

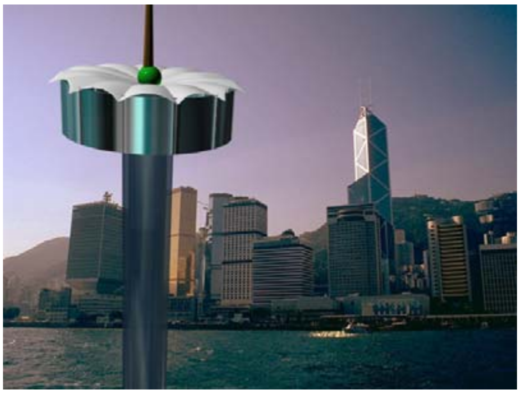
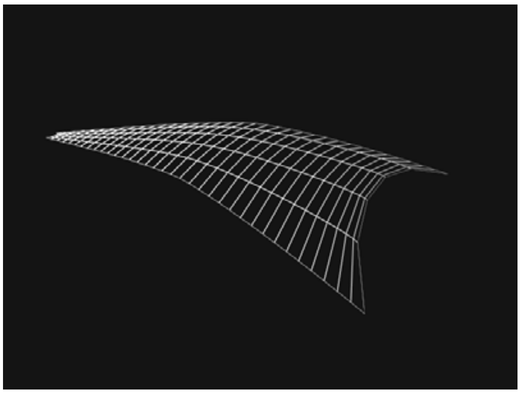


Figure 1



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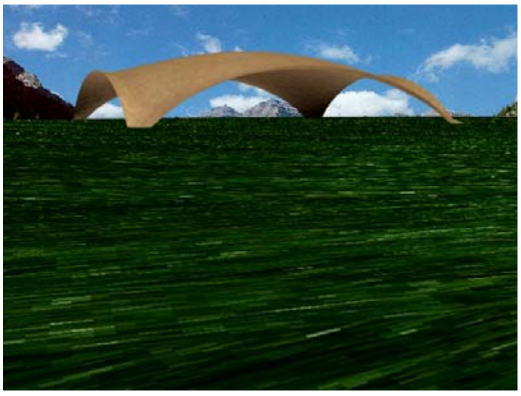

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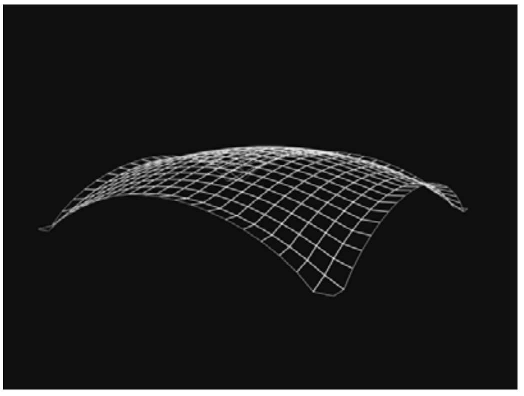
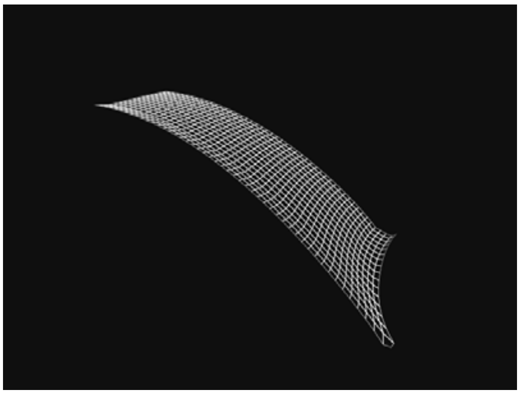


Figure 4

Figure 5



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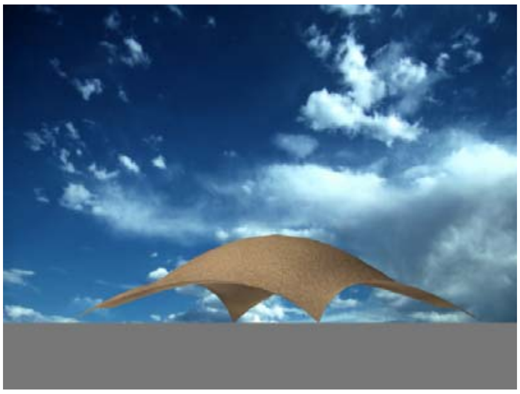

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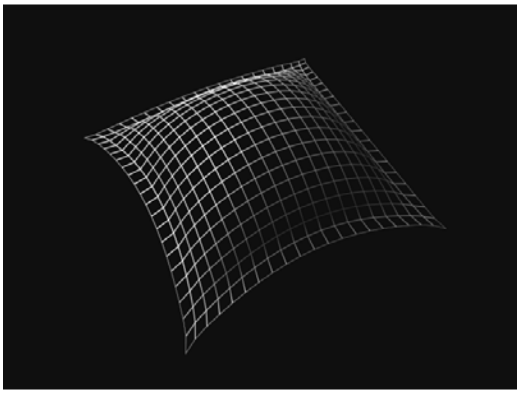

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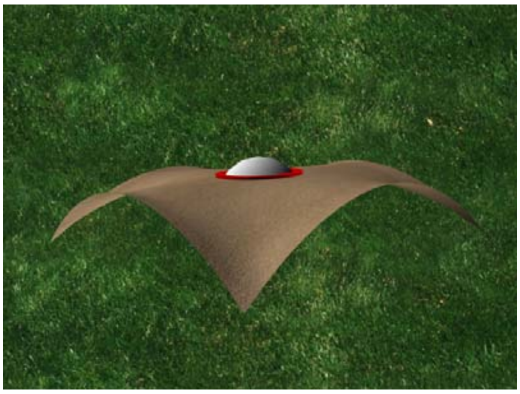

Figure 9

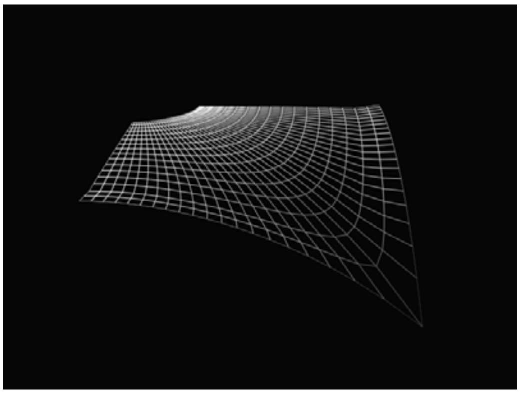

Figure 10

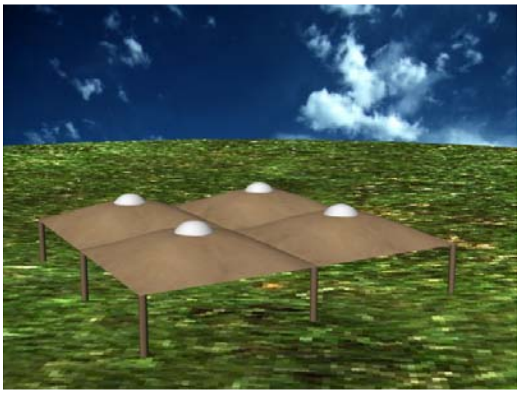

Figure 11

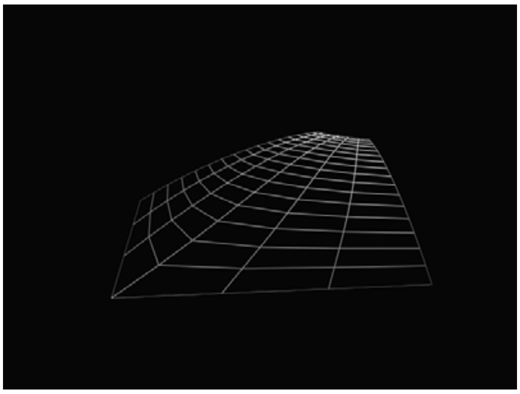

Figure 12

Figure 1 shows the image of a ring-shaped shell of pneumatic type which is modeled on Isler's shell for a large warehouse. Figure 2 shows the formfinding stage for one leaf of the shell. The Young's Modulus is increased in the elements along the two radial edges. This makes these edge elements more stable. Then the curvature of the shell is increased near the edge.

Figure 3 and Figure 7 show the images of two square shells. Figure 4 and Figure 8 present the formfinding process. In order to obtain anti-curvature on the edges some edge nodes are given additional nodal forces in the vertical direction.

Figure 5 shows the image of a hall for a tennis pavilion in which four elements of $16 \times 48(m)$ are joined. Figure 6 gives its formfinding. Because of the symmetry only one quarter of a shell is modeled. Some fine adjustments are required on the edge elements. A sufficiently doubly curved shell with sound and distinct counter-curvatures at the free edges usually is able to cope with instability and to carry all load cases.

Figure 9 shows the image of a roof of a swimming pool. This shell has free edges and is supported on four points on a square plan. Figure 10 presents the formfinding stage of the roof. Due to symmetry only a quarter of the shell need be simulated. As described above, from the flat position of the mesh generation, a uniformly distributed load or internal inflation pressure is applied and a geometric non-linear analysis is processed until the required height for the structure has been reached. To provide edge kick up, nodes on the edges are subjected nodal loads, some nodes along the very edge are set with upward forces while downward forces are set to some nodes where the curvature should be changed by observing Isler's shell.

Figure 11 shows the image of a block of four bubble shells of $20(m)$ span covering an area of $1600(m^2)$. Figure 12 presents a quarter of the computer model for formfinding of a bubble shell.

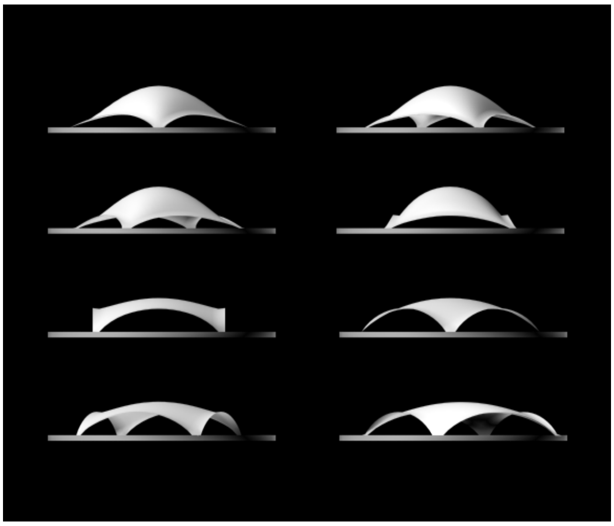

Figure 13 shows two examples of shells (upper four and bottom four) from different angles. They change their visual appearance totally when seen from the varying view. So one can see that shell structures may, more than other buildings, offer a great variety of changing aspects.

It should be seen that computer models have similar characteristics to physical models in (1) They are three-dimensional; (2) They are analyzed by setting real material; (3) They are visual.

Figure 13

Compared with physical model

1. The computer model is simple. Architects and engineers who have engineering background and some computer knowledge can easily develop these models whereas the process of physical modelling is not so easy to master. It requires experimental work of quite high accuracy.
2. The computer model can save time and energy. It is possible to adjust and modify parameters thus giving many possible shapes to view.
3. Due to the reasons above it seems the computer modelling method has some distinct advantages.

Once the desired membrane surface has been obtained, the shell thickness and concrete material property are added to form the real shell. To verify the mechanical behavior of the concrete shell structure with the shape obtained, the finite element method is employed for the calculation of stresses and displacements of the shell. The results we obtained so far are satisfactory. As Heinz Isler once said [7], all our computer programs are unproven intellectual fictions until they are proved by physical reality and monitored over a period of time. By comparing with his work we bring reality to our fiction.

4. Conclusions

1. Formfinding is one of the most important factors in shell design. Computer modelling shows a simple, quick and flexible way to do this. Much of the shell's success is a function of an interplay of aesthetics and mechanics.

2. Computer shell modelling is easy to undertake and also is a sound and creative counterpoise to the physical modelling.

3. The deformation patterns calculated on models might be a criterion for qualification of the shapes. Thus additional work has to be done to check the shape performance by studying geometric non-linear analysis of the real shell.

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