Formfinding, Analysis and Patterning of Regular and Irregular-Mesh Cablenet Structures

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ABSTRACT

This paper deals with the design of lightweight surface-stressed structures incorporating doubly-curved cable nets such as the Montreal Expo and Munich Olympic Stadium roofs from the 1960's and early 1970's respectively. The problem of form-finding and patterning cable net structures is quite different to that of textile membrane surfaces, especially in the case of structures fabricated with a constant mesh size. This paper explains, with specific reference to the **Easy** lightweight structure design system, how to deal with both regular mesh and irregular mesh structures.

KEYWORDS

Form-finding, Cutting-pattern, Lightweight-structure, Surface-structure, Tension-structure, Cablenet, Membrane, Aviary, EASY, CAD/CAM, Roof.

INTRODUCTION

This paper deals with the design of lightweight surface-stressed structures incorporating doubly-curved cable nets. Among the more famous examples of such a structural configuration are the Montreal Expo and Munich Olympic Stadium roofs from the 1960's and early 1970's respectively. Since that time most of the major stressed surface architectural structures have utilised textile membranes rather than cable nets. Over the past decade there has been a small resurgence in the development of cable net designs for niche areas, particularly aviary projects and the reinforcement of large span pneumatic structures. The problem of form-finding and patterning cable net structures is quite different to that of textile membrane surfaces, especially in the case of structures fabricated with a constant mesh size.

Figure 1: *Munich Olympic stadium roof during construction.*

STRUCTURAL ANALYSIS OF CABLENET STRUCTURES

Cablenet structures typically experience relatively large deflections when subjected to applied load. It is therefore necessary to use a geometrically non-linear structural analysis system. The non-linear analysis module of the **Easy** system uses a *Conjugate Gradient* solver. A feature of this analysis tool is its pre-conditioning technique which uses a force-density based formulation to provide extremely robust and stable performance. Quick convergence is achieved from even grossly out of position starting geometries, and without the necessity of judicious manipulation of weighting parameters. This greatly facilitates the preparation of partially stressed erection sequence analyses. Report files listing the cable forces and nodal deflections are generated. Additionally, interactive *Windows 9x/NT* based graphical tools allow the user to quickly identify response patterns and focus on the critical areas.

FORMFINDING AND PATTERNING OF CABLENET STRUCTURES

In the case of textile or isotropic membrane surface structures, the task of form-finding the surface can, and should, be separated from that of generating the cutting pattern. With cablenet structures the relationship between the two processes is much more closely linked.

It is important to distinguish between the two main types of cablenets used in architectural structures. *Regular-mesh* nets are commonly defined as those which have constant connector to connector link lengths, as well as those which have constant lengths in each of two directions.

Irregular-mesh nets are usually defined as those which have variable link lengths. On this basis a completely symmetrical radial mesh which appears geometrically regular would be termed irregular. This is appropriate since classification is largely made from an industrial fabrication point of view. Although partially geometrically regular, unlike square or rectangular topologies, radial nets are not automatically fabricated. Some irregular mesh structures are shown in Figures 2-4.

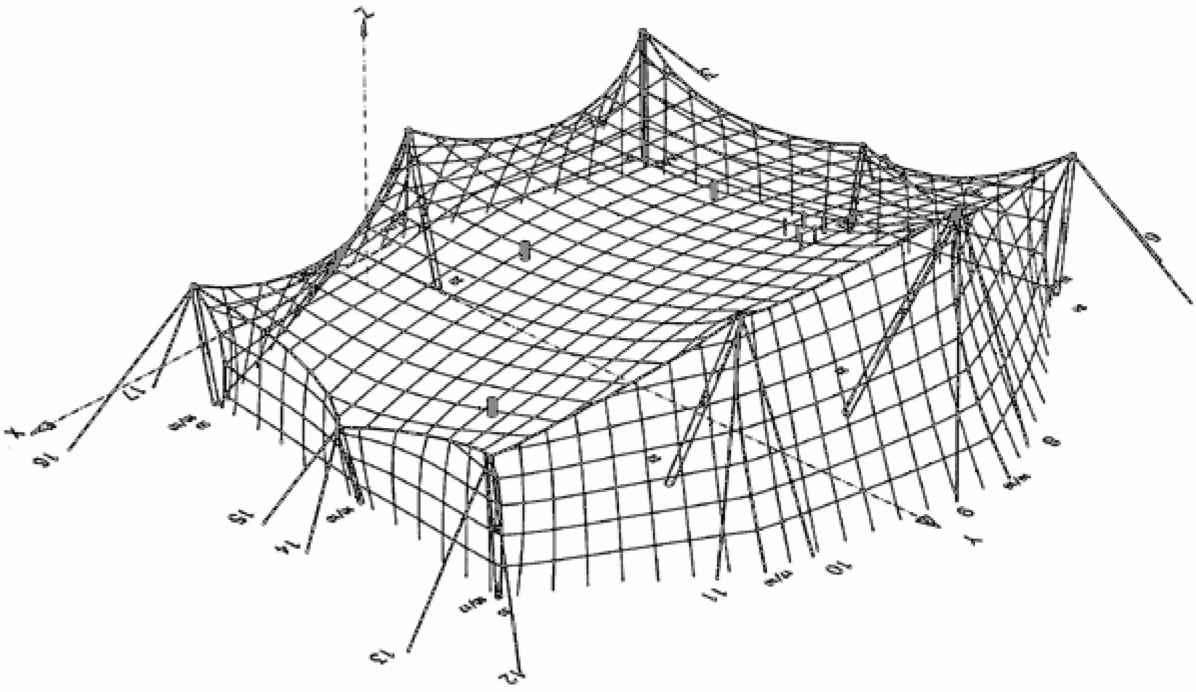

Figure 2: *Berlin zoo aviary cable net structure.*

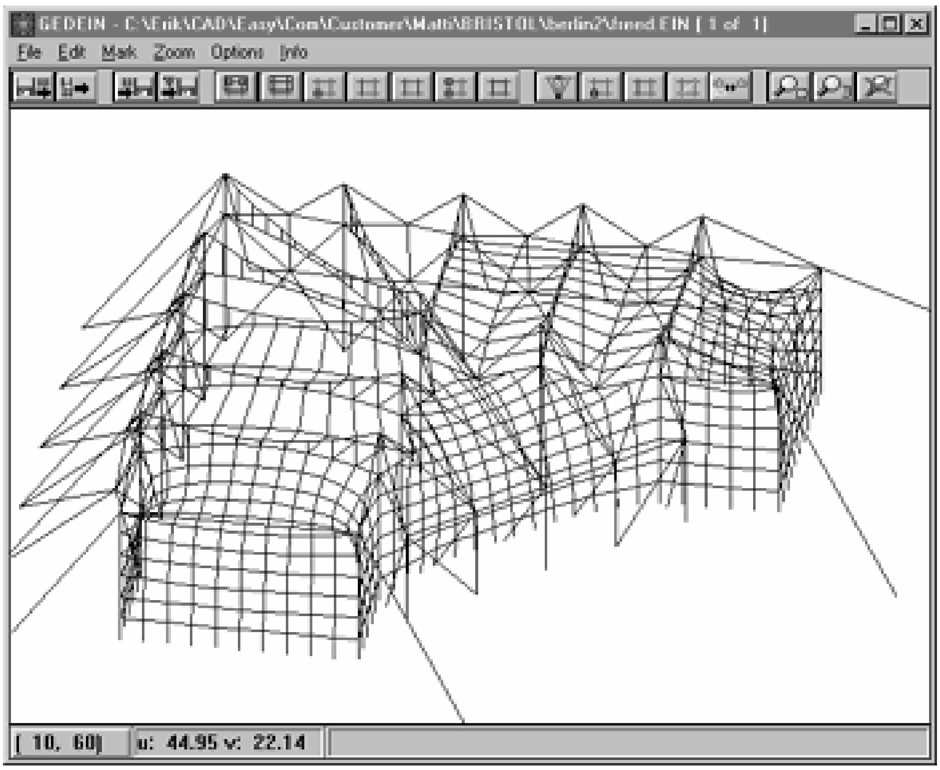

Figure 3: View of Bristol zoo aviary.

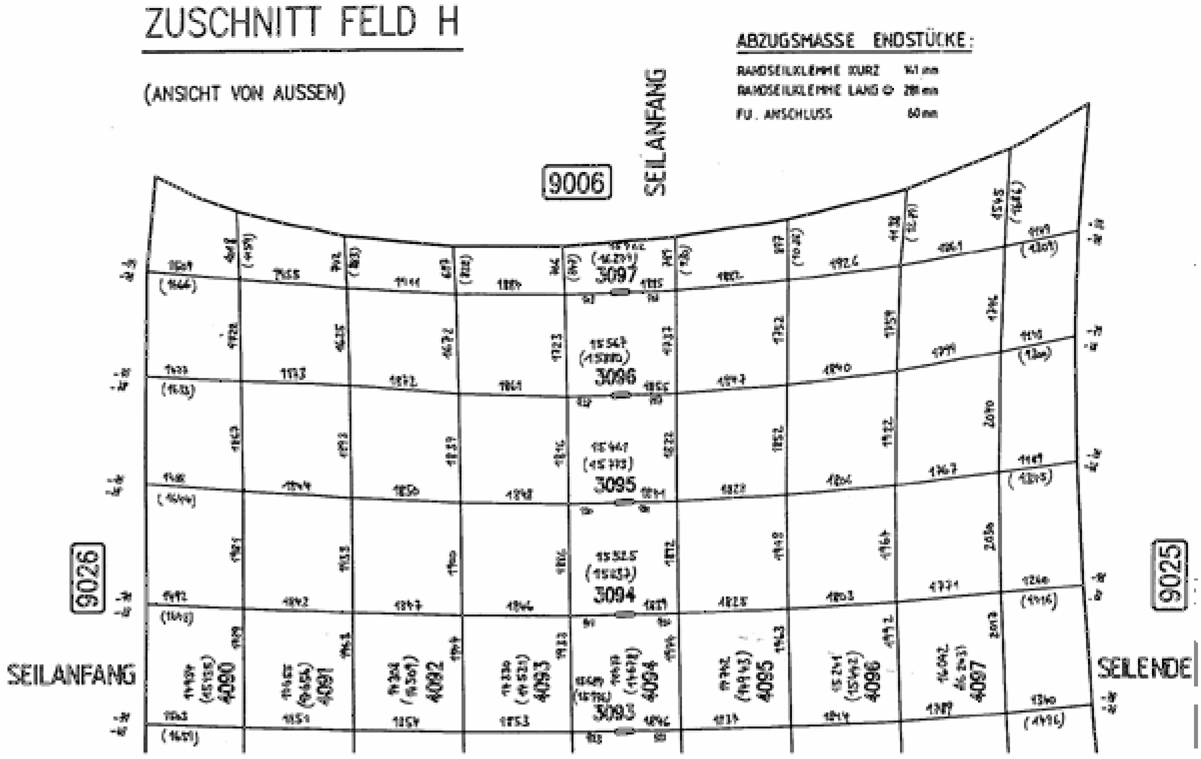

Figure 4: Cutting pattern for part of Berlin zoo aviary cable net.

Irregular-mesh cablenets

The approach used in the **Easy** system for the form-finding of irregular-mesh cablenets is the same as that used for textile roof form-finding. Having specified a mesh in terms of link topology and force-density, a force equilibrant configuration is determined through the solution of the resulting linear force-density system. This solution is then checked for suitability, modifications are made to the mesh, and the design iteration repeated. When the configuration is finalised, the cutting pattern data for the cables and cladding is determined using the stressed geometry together with the material's elastic properties.

Regular-mesh cablenets

Compared to irregular-mesh nets, regular-mesh designs are more difficult to design. In the case of regular-mesh nets most of the link lengths must, by definition, remain fixed. The form-finding and patterning task therefore becomes one of determining the shape of each unstressed planar net area. Depending on the particular geometry of the desired surface, two approaches to this problem may be taken with the **Easy** system. These are *Mixed Element Formfinding* and *Contacting Surface Growth.*

Mixed Element Formfinding with Surface Flattening

In pure force density form-finding each link element's force is determined from the specified force density together with the end node geometry. Similarly, in the case of elastic analysis, each elemental force is a function of the link stiffness, unstressed length and end node geometry. By using both force density and elastically controlled links in the same analysis, it is possible to constrain particular link lengths. This technique is most commonly used to introduce fixed length support structures such as masts and struts. The usual objective being to determine equilibrant structural configurations with unstayed mast tops.

In order to ensure that a form-found cable net may be fabricated from regular mesh, it is necessary to elastically control all inner links and force density control those connecting to the boundary cables.

Clearly it is necessary that for a cable net configuration to be acceptable all links must be neither over nor under stressed. Depending on the particular configuration, it may or may not be possible to achieve the desired design. Complications also arise in the specification of the basic net topology. *Mixed Element Formfinding* uses a fixed net topology and only varies the lengths of the links which connect the regular inner area to the boundary. Therefore, there is an inherent limitation in the amount of variation each link can experience. It is usually a good idea to pre-process meshes in order to ensure that none of the variable-length links have very short starting lengths. It must be emphasised, however, that simply erring on the side of extra length for these links will result in extremely ugly structures. Since the inner mesh is by definition regular, it is important that the connection to the boundary be achieved in as smoothly flowing a manner as possible.

To achieve the best results it is normal to adopt a double cycle iterative design procedure. First a cable net mesh is generated within planar boundary curves.   This is then form-found with only the lengths of the links connecting to the boundaries as variables. At this stage variations are made to the link force densities and the form-finding repeated. Usually it will become quickly apparent that along some boundaries the links are too long or there are sharp angular deviations. On the basis of the feedback obtained, the original mesh topology is then modified by adjusting the planar mesh boundary.

A particularly good strategy for improving the quality of the initial mesh is to use *Surface Flattening*. In this technique the structure is first form-found using conventional force density form-finding. Each cable net section is then extracted and flattened subject to constraints on both length and angular distortion. Since each starting surface is already irregular, and extra distortion is introduced by the flattening, the resulting net geometry is of course unsuitable for fabrication. By extracting the planar boundary and then remeshing within it, very good starting meshes can be generated for *Mixed Element Form-finding*

Contacting Surface Growth

One of the biggest differences between membrane and cable net structures lies in the area of shear distortion. In the case of a membrane it is important to orient the individual cloths with the principal surface curvature, and therefore with the principal stresses. Significant shear will generally result in wrinkling which is both visually unattractive as well as a potential problem for the fabric. With cable nets this is not the case. Indeed it is through the progressive modification of the inter-cable crossing angles that good double curvature can be realised from planar net.

It should be clear that having specified the position of a cable net node on a surface, the positions of the neighbouring nodes are constrained to lie on the curve defined by the intersection of the surface and the sphere of mesh width radius centred at the constrained node. Specifying the positions of two of a nodes neighbours will therefore limit that nodes position to at most two possible positions.

It is possible to use this property to grow regular-mesh nets over general surfaces. Starting by fixing a line of net links on a symmetry line, or other central position, the positions of the neighbouring nodes can be progressively determined on the basis of sphere surface intersection. Finally the surface boundary can be used to clip the generated net.

In the specialist contacting module of the **Easy** system it is possible to perform both force density form-finding and elastic analysis subject to highly sophisticated nodal constraints. Rather than merely having the ability to constrain nodes to lie on the main *x*, *y* or *z* axes planes, it is possible to constrain to any arbitrary plane or line.

The possibility therefore exists to perform analysis of a net subject to it being in contact with a doubly curved surface. First a good equilibrant surface is form-found using conventional force density form-finding. A regular planar mesh with elastically controlled links is then dropped onto the surface. All nodes are constrained to lie on the surface with one node additionally fully fixed. At least one other node must be partially restrained to lie on a plane. During the subsequent elastic analysis the cable net nodes will slide over the surface and attain equilibrium.

EXAMPLE: FORMFINDING OF REGULAR MESH CABLENET

The stages in the formfinding and simultaneous patterning of a regular mesh cablenet structure are shown in Figures 5-11 below.

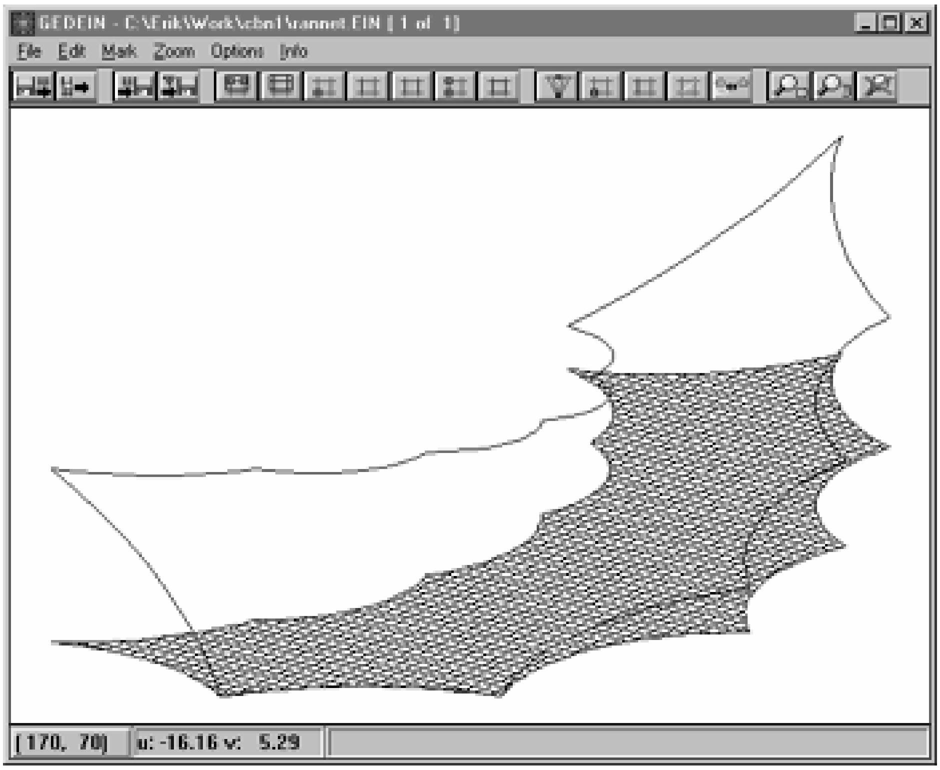

Figure 5: *Perspective view of 3D desired boundary curves together with the initial planar mesh generated by clipping to the default construction plane.*

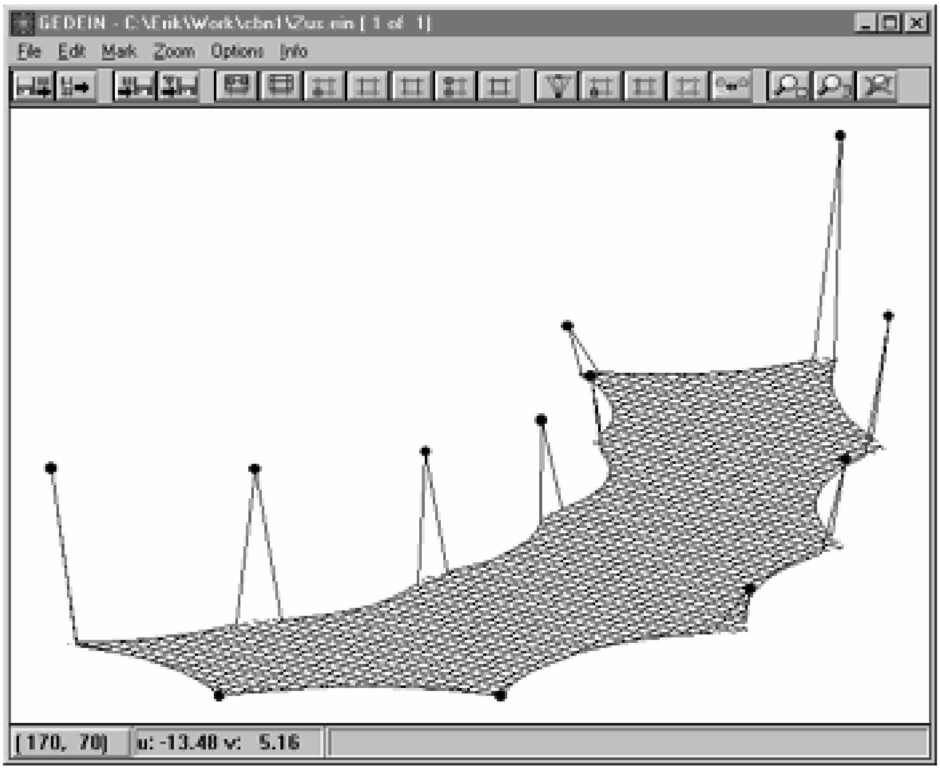

Figure 6: Perspective view of initial mesh with fixed points specified ready for force-density form-finding.

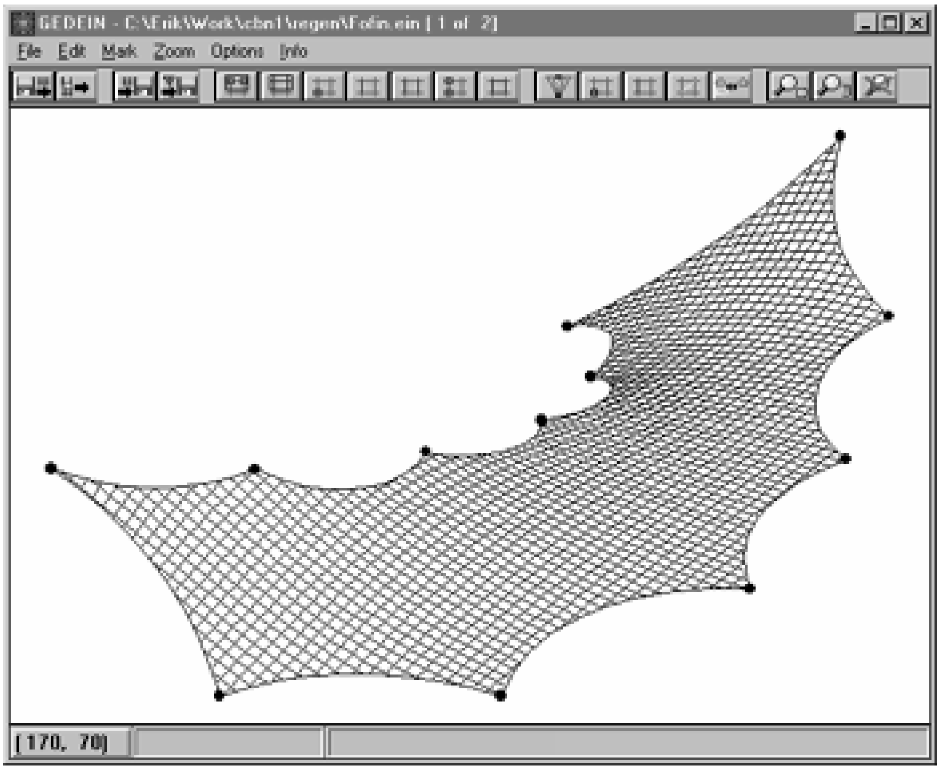

Figure 7: Perspective view of force equilibrant irregular mesh form-found surface.

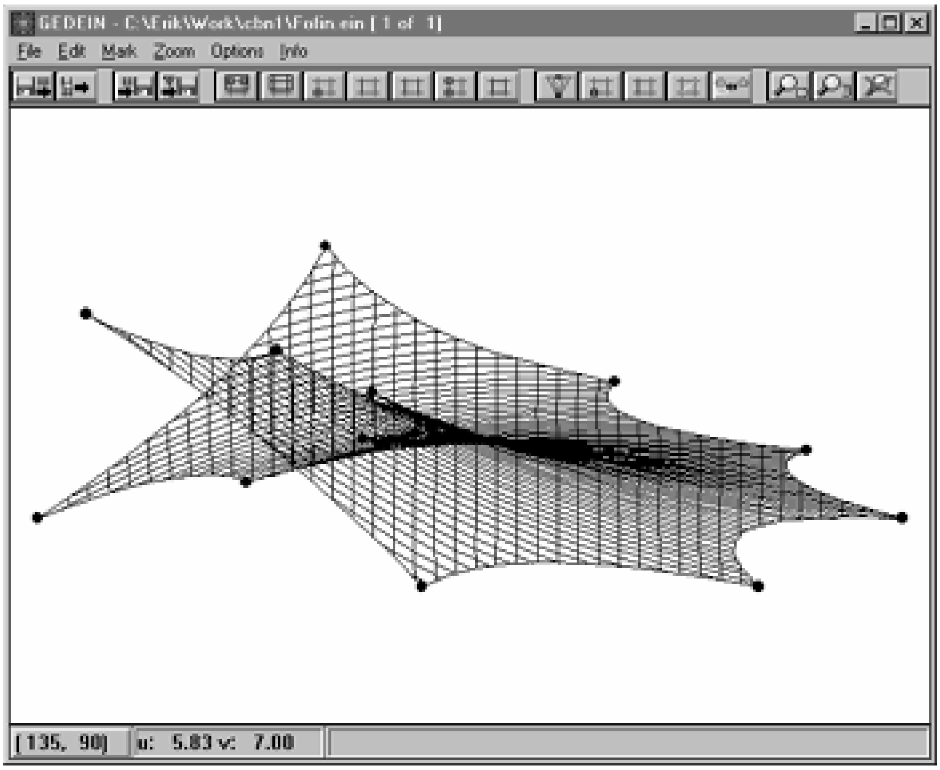

Figure 8: Side view of force equilibrant irregular mesh form-found surface.

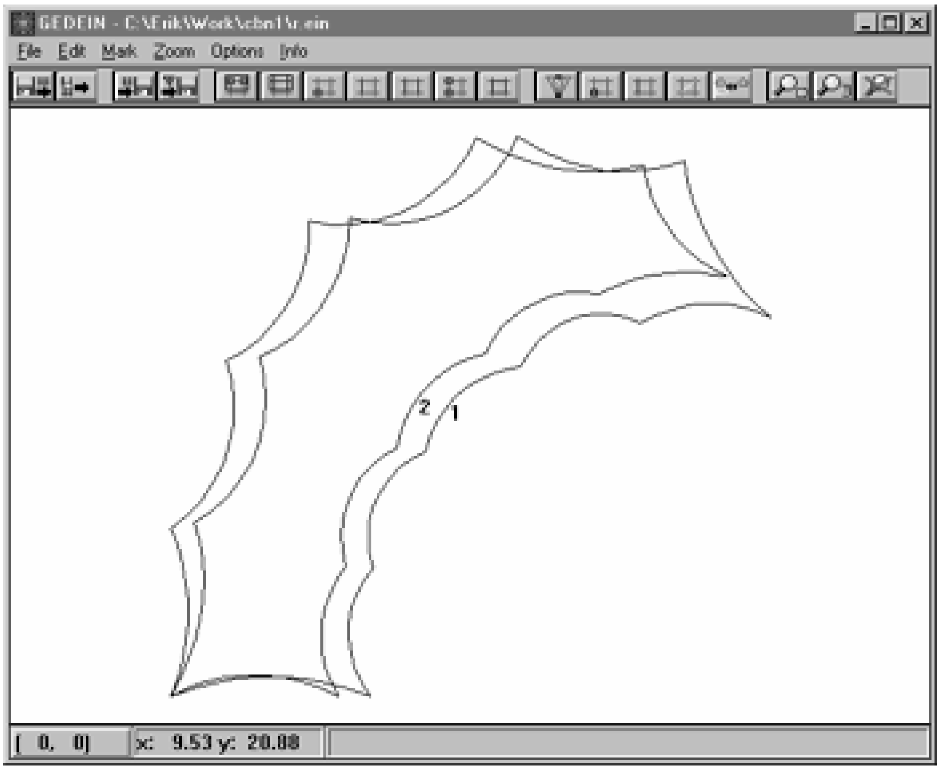

Figure 9: Plan views of (1) improved boundary extracted from flattened irregular mesh surface and (2) projection of initial desired boundary curves.

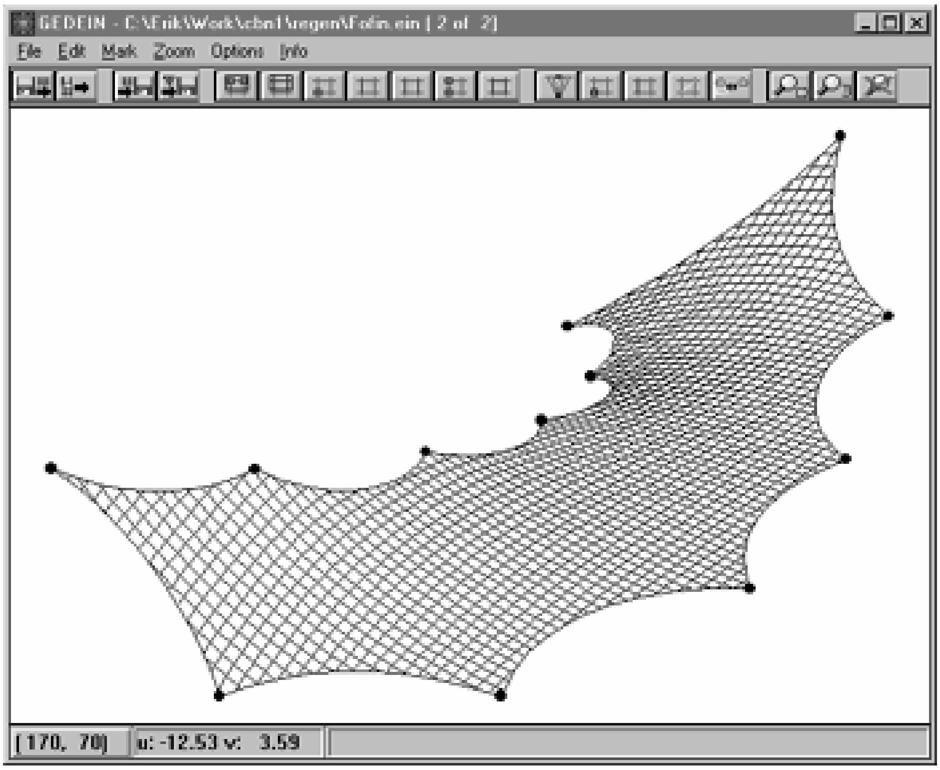

Figure 10: Perspective view of force equilibrant regular mesh form-found surface.

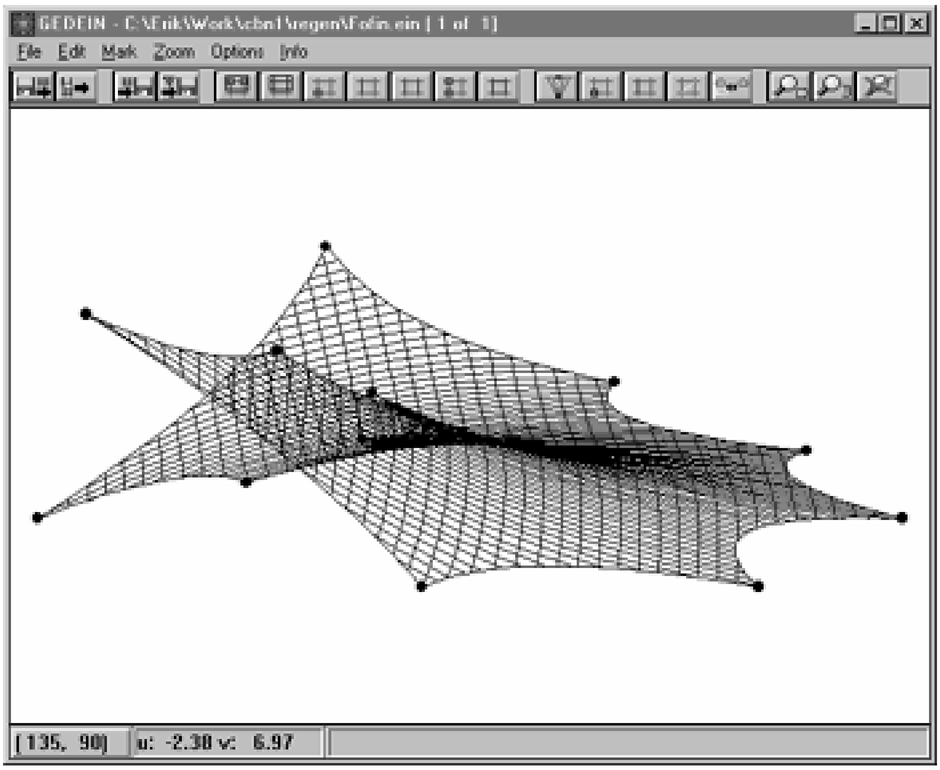

Figure 11: Side view of force equilibrant regular mesh form-found surface.

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