Dynamic Analysis of a Pendulum with Two Cables

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Introduction

Lighting equipment or sound speaker systems are hanged from the ceiling in many dome structures or gymnasia. They are clamped with slender members like rods or cables. Recently such instruments tend to be bigger and heavier, and their fall may cause serious damage. Then it is important to clarify dynamic behavior of the above mentioned structures under cyclic loads like earthquakes. Two conditions of the tensile and the compressive may be repeated in slender members(called cables below). Generally cable members easily buckle under slight axial compression forces and this phenomenon is called the loosening. When tensile axial forces act on these cables which have loosen once, tensile stresses occur in cables as soon as being in tightening. It is said that such tensile stresses at this moment are like impact. But there are a number of unclarified points regarding the dynamic behavior of such structures in cyclic loads. The present report therefore aims at presenting basic data on designing these structures via the simplest model's results which is calculated by the finite element method with consideration of the global geometrical nonlinearity and material nonlinearity to evaluate cable's loosening.

Analytical model and numerical method

Fig.1 shows the analytical model considered in this paper. This model is one of the simplest for equipment hanging from ceiling and is a pendulum which consists of single mass node and two cable members. We calculate this model with the finite element method.

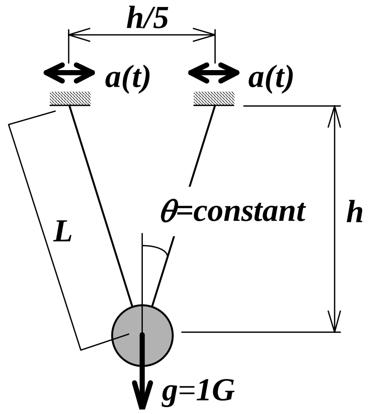


Fig.1 *Present Analytical Model*
Pendulum Model with Single Mass Node and Two Cable
(Half open angle; θ=constant)

The assumptions on numerically modeling are followings.

i) *Cable's mass density is ignored*. Then one truss element for one cable is used.

ii) *Loosening behavior of cables under compression is evaluated with material nonlinearity*. Namely cable's material property is one way stress condition as **Fig.2**.

iii) *This system is considered as 2-dimensional problem*. The total freedom is only two of horizontal displacement and vertical displacement.

The aim to consider the above assumptions is to clarify the macroscopic and essential behavior of this structural system.

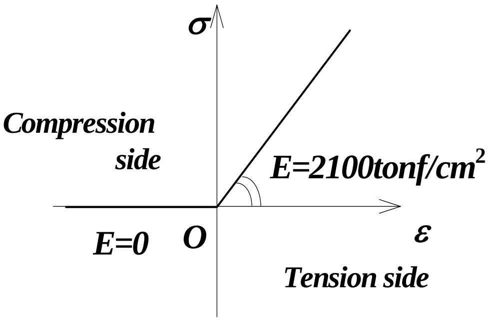


Fig.2 *Material property of cable*
E:Young's modulus

The applied numerical method is based on the below concept.

1) Geometrical nonlinearity is formulated with *the updated Lagrangian formulation*. The Green strain and 2nd Piola-Kirchhoff stress is taken account.

2) *The Newmark method* with β=1/4 and γ=1/2 in Eq.(2) is applied to time integration scheme.

Motion equation;

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{u}} + \boldsymbol{C}(t)\Delta\dot{\boldsymbol{u}} + \boldsymbol{K}(t)\Delta\boldsymbol{u} = \Delta\boldsymbol{f}_{ex} + \boldsymbol{R}(t), \quad \boldsymbol{R}(t) = \boldsymbol{f}_{ex}(t) - \boldsymbol{M}\ddot{\boldsymbol{u}}(t) - \boldsymbol{C}(t)\dot{\boldsymbol{u}}(t) - \boldsymbol{f}_{in}(t) \tag{1}$$

Applying Newmark method to this equation,

$$\left[\frac{1}{\beta\Delta t^2}\boldsymbol{M} + \frac{\gamma}{\beta\Delta t}\boldsymbol{C}(t) + \boldsymbol{K}(t)\right]\Delta\boldsymbol{u} = \boldsymbol{R}_{eff}(t) \tag{2a}$$

$$\boldsymbol{R}_{eff}(t) = \Delta\boldsymbol{f}_{ex} + \boldsymbol{M}\left\{\frac{1}{\beta\Delta t}\dot{\boldsymbol{u}}(t) + \frac{1}{2\beta}\ddot{\boldsymbol{u}}(t)\right\} + \boldsymbol{C}(t)\left\{\frac{\gamma}{\beta}\dot{\boldsymbol{u}}(t) + \frac{\Delta t}{2}\left(\frac{\gamma}{\beta} - 2\right)\ddot{\boldsymbol{u}}(t)\right\} + \boldsymbol{R}(t) \tag{2b}$$

where $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ are mass matrix, damping matrix and stiffness matrix, $\boldsymbol{R}, \boldsymbol{f}_{ex}, \boldsymbol{f}_{in}$ are unbalance force, external force and inner force vector, $\ddot{\boldsymbol{u}}, \dot{\boldsymbol{u}}, \boldsymbol{u}$ are acceleration, velocity, displacement vector respectively.

The value of time increment is equal to T_k/70 in this paper. T_k is the eigen period for the longitudinal vibration mode of a cable, namely $T_k = 2\pi\sqrt{\frac{ML}{EA}}$, M; mass, A; cable's section area. This time increment value becomes very small at the actual calculation. Furthermore The eigen period T_k means the highest eigen period for the present model. Generally speaking, the value of a time increment on applying the Newmark method is maybe about T_1/20; T_1 is the lowest eigen period since higher mode has no influence on the global behavior and is dissipated with damping effect. However, the longitudinal vibration mode has a great influence in the case of this structural system as described later. Thus such value is applied. Solving Eq.(2), Newton Raphson method is applied and the convergence is estimated by the ratio of the norm of unbalance force vector to the norm of inner force vector. The tolerance is equal to 10^{-12}.

3) Rayleigh damping is applied.

$$\boldsymbol{C}(t) = \alpha_0 \boldsymbol{M} + \alpha_1 \boldsymbol{K}(t) \qquad (3)$$

It is very difficult to predict the damping effect. Here, we assume that α_0, α_1 in Eq.(3) is determined using 2% at two specific frequency: (1) the lowest frequency(f'_1) of the model in which loosening is not considered; (2) 1000 times f'_1.

Loading condition is as following.

Step 1: The gravity acts on the mass node statically.

Step 2: The 1995 Hyougo-Nanbu (Kobe, Japan) earthquake wave (NS-component, Maximum acceleration:818cm/sec^2) in **Fig.3** is loaded as cyclic loads. *This loading is inputted as constraint displacement at fixed ends of both cables*, not at the mass node directly.

It is the reason to input earthquake load from the fixed end side that the mass node may float since both cables loosen simultaneously. It is obvious that the acceleration of any earthquake do not act on the floating mass node.

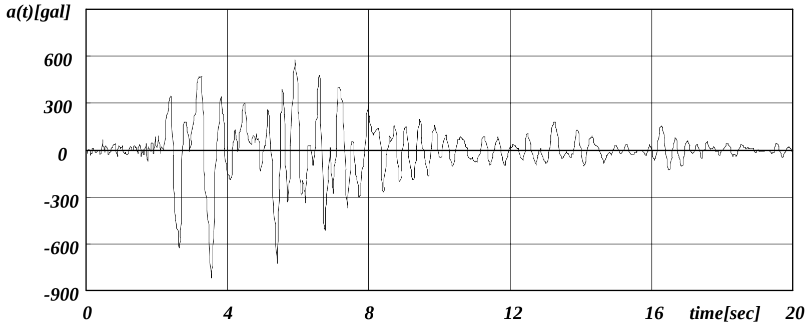


Fig.3 *Acceleration time history for Hyougo-Nanbu earthquake*

Numerical results for undamped model

In this section, we describe the characteristics of the obtained results for a typical undamped model with M=1.0x10^{-5}(tonf/cmsec^{-2}), A=0.1(cm^2), L=100(cm). The horizontal and vertical displacement time history is shown in **Fig.4(a)**. Each value is normalized by the length of L. The maximum response of horizontal displacement is about 0.5 and vertical one is about 0.3. The period is about 2 seconds and this value is different from the period as a three hinged structure, which is the model without loosening, but equal to the period T_p as a 'Pendulum' with length of L under the gravity, namely $T_p = 2\pi\sqrt{\frac{l}{g}}; g = 980cm/\sec^2$. This matter means that one cable is always loosening.

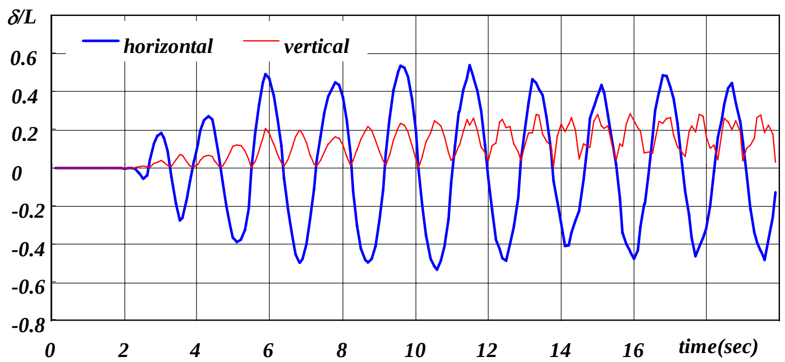


Fig.4(a) *Displacement Time History*

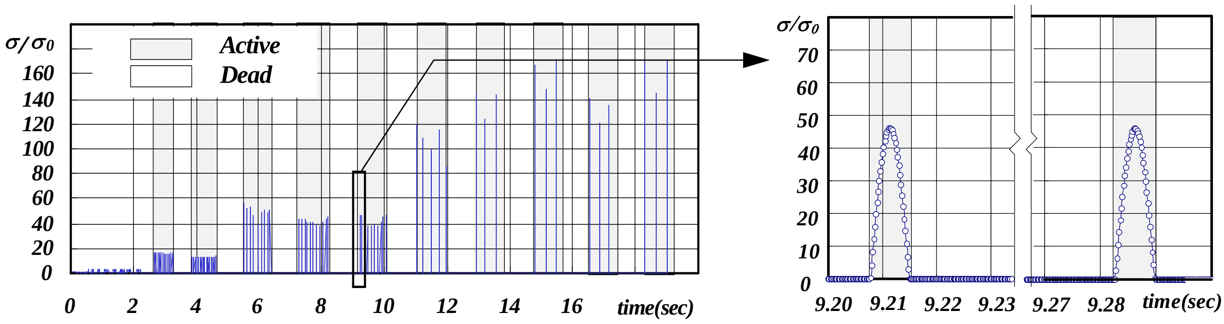


Fig.4(b),(c) *Axial Stress Time History in a Left-side Cable*

But, these responses are not as simple as one of a 'Pendulum' after 10 seconds particularly. It is obvious that the higher vibration mode has an influence on this system.

The axial stress response of a left-side cable is shown in **Fig.4(b)**. Of course, one of a right-side cable is similar macroscopically. These are normalized by the stress value at the first step, when acts only the static gravity. In this figure, the notation of 'Active' or 'Dead' means that this cable should be in tightening or in loosening on the consideration of this model as a 'Pendulum'. This figure leads to the followings: At first, the region of 'Dead', any stress does not respond. This matter means the cable's loosening behavior is evaluated exactly, and it is quit natural. On the other hand, the response at the region of 'Active' is not smooth and seems to be discontinuous. Such a result can not be anticipated on physical grounds. Because the tensile axial stress in tension-side cable should be always constant if we assume that this system behaves as a 'Pendulum' after compression-side cable's loosening. Then we enlarge the part surrounded by Box and show it in **Fig.4(c)**. Such an enlargement unable us to understand the characteristics of the behavior of this system. Concretely speaking, the axial stress changes continuously during the very short term and the period of a axial stress is nearly equal to 0.015 second and this value corresponds to T_k, which is the eigen period for longitudinal vibration mode of cables. Then the tension-side cable may even become to be in loosening due to such a vibration mode. In fact, true 'Dead' zone exists in **Fig.4(c)**. Furthermore, it is clear from **Fig.4(d)** too. This figure shows the motion of the mass node. The dotted lines is circular arc lines with the radius of L around each supported point. Considering this system as a 'Pendulum', the mass node has to move along the two dotted lines. However, in the present result, however the mass motion is complex. Obviously the mass node moves not only in the lateral direction but also in the longitudinal direction. Such a longitudinal vibration cause the

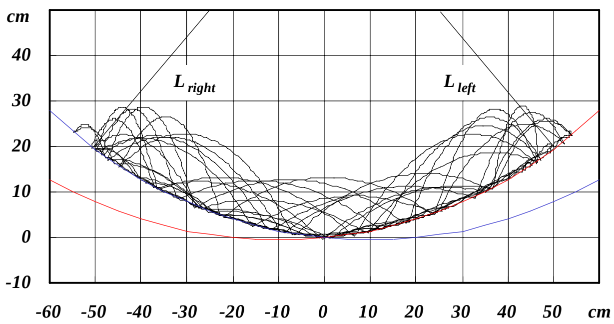


Fig.4(d) *Orbit of a Mass Node:*
Solid line; Numerical results
Dashed line; circular arc lines with the radius of L around each supported point.

quite large axial stress as shown in **Fig.4(b)** like acting a impact load.

Influence of two eigen period T_k and T_p in undamped model

In the previous section, we clarified T_k and T_p have a great influence on this system. Thus, aim to examining the influence in detail, we calculated this structural system with various *M*, *A* and *L*, See **Table 1** and **2**).

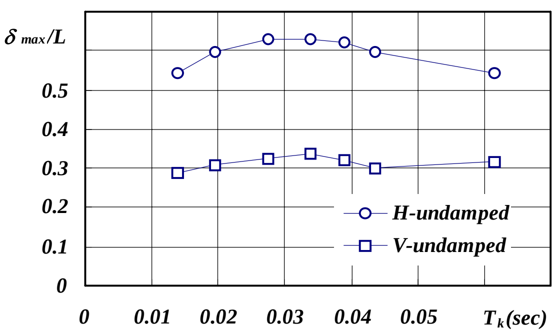
Model Name	*A* (cm^2)	*L* (cm)	*M* (tonf/cmsec^{-2})	T_k (sec)	T_p (sec)
A01L100M1	0.10	100.5	1.0×10^{-5}	0.014	2.0
A01L100M2	0.10	100.5	2.0×10^{-5}	0.019	2.0
A01L100M4	0.10	100.5	4.0×10^{-5}	0.027	2.0
A01L100M6	0.10	100.5	6.0×10^{-5}	0.034	2.0
A01L100M8	0.10	100.5	8.0×10^{-5}	0.039	2.0
A01L100M10	0.10	100.5	1.0×10^{-4}	0.043	2.0
A01L100M20	0.10	100.5	2.0×10^{-4}	0.061	2.0

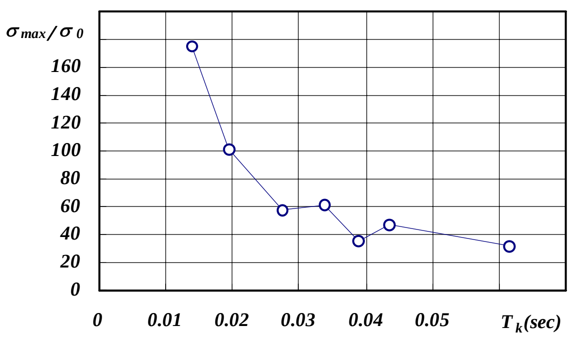
Table 1 *List of Analytical Model*
- T_p is constant -

Model Name	*A* (cm^2)	*L* (cm)	*M* (tonf/cmsec^{-2})	T_k (sec)	T_p (sec)
A01L100M1	0.10	100.5	1.0×10^{-5}	0.014	2.0
A015L150M1	0.15	150.8	1.0×10^{-5}	0.014	2.4
A02L200M1	0.20	201.0	1.0×10^{-5}	0.014	2.8
A04L400M1	0.40	402.0	1.0×10^{-5}	0.014	4.0
A08L800M1	0.80	804.0	1.0×10^{-5}	0.014	5.7

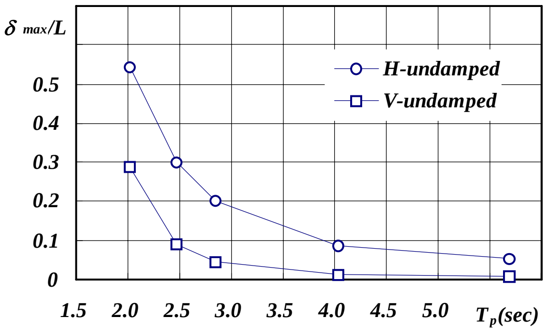
Table 2 *List of Analytical Model*
- T_k is constant -

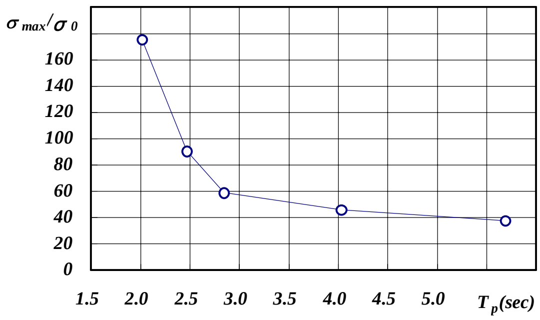
At first we explain the results in the case of T_p=constant=2.0sec. **Fig.5(a)** shows the maximum displacement response spectrum of the mass node in both the horizontal and vertical direction changing T_k. Similarly **Fig.5(b)** shows the maximum axial stress response spectrum. Though the maximum response spectrum of stress should correspond to one of displacement in the case of a linear problem, we can not confirm such correlation between this two figures. Namely the maximum displacement responses hardly change while the maximum stress response decreases considerably as increasing the value of T_k. These results mean that the global motion like a 'Pendulum' depends on T_p and the maximum value of a stress does not become to be great whenever the global motion is large. Because it is the severe longitudinal vibration that causes the great value of stress as mentioned via the example in the previous section. Furthermore such vibration is considered to depend on T_k.





From the facts described above, it is obvious that only T_p has an influence on the maximum





displacement response. However, we can not conclude that the maximum stress response depends only on T_k via these results directly. Thus, we examine the effect of T_k for the mechanical properties

Fig.5(a) *Maximum Displacement Response* **Fig.5(b)** *Maximum Stress Response*

Fig.6(a) *Maximum Displacement Response* **Fig.6(b)** *Maximum Stress Response*

with the series of analytical model as shown in **Table 2**. These models have a constant(=0.014sec) of T_k and various value of T_p. The maximum displacement response spectrum is shown in **Fig.6(a)** and one of stress in **Fig.6(b)**. In this case, the stress response decreases corresponding to the displacement response.

The present results and the results shown in **Figs.5** is reduced to the followings.

(a) The maximum displacement response depends on only the eigen period considering this structural system as a ‘Pendulum’.

(b) The maximum stress response depends on the above mentioned eigen period and the eigen period for the longitudinal vibration mode.

It is an important problem how the two periods have an influence on the stress response. In this paper, we introduce the new parameter α to estimate the combination of these periods.

$$\alpha = \sqrt{\frac{v_{\max}}{L} \cdot \left(\frac{T_p}{T_k}\right)}, \qquad \text{where} \quad v_{\max} := v_{\max}\left(T_p\right) \tag{4}$$

This parameter can be led by the consideration of the change of momentum at the cross point of two dashed lines in **Fig.4(d)**. Function v_{max} is determined with the results in **Fig.6(a)**

Fig.7 shows the relation between and the maximum stress response and the new parameter α. Generally the strongest correlation was observed between both although the plots in the graph are slightly scattered and they have a simple proportional relation.

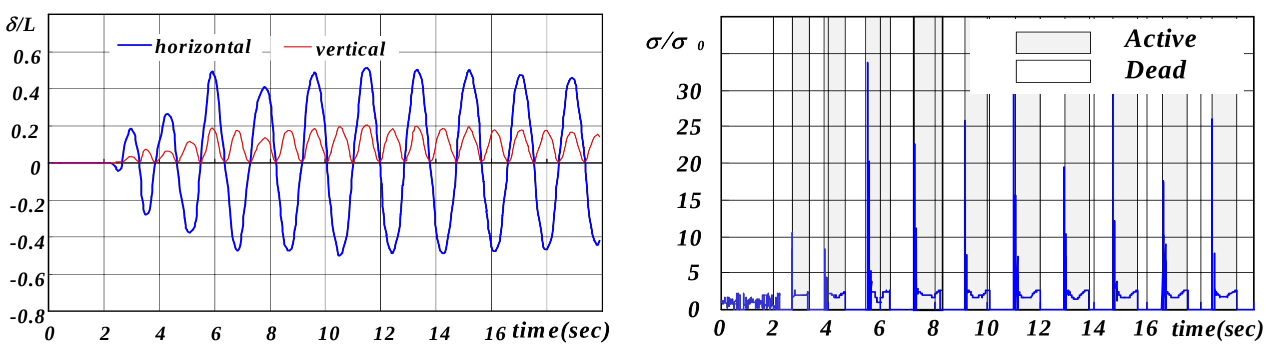
It is clear from this figure that the stress response is evaluated with the present parameter macroscopically .

Influence of damping effect

In this section, we examine the damping effect on the behavior of this structural system. The results of the numerical example which has the same configuration as the model in **Figs.4**, is shown in **Figs.8**. Compared with the results without damping effect, it is clear from these figures that these results have the characteristics as follows.

a) The global motion of the vertical and horizontal displacement is as same as one of an undamped model. This reason is that the damping effect is evaluated by Eq.(3) and the eigen frequency for this vibration mode is too low for the damping effect to have an influence on this behavior.

b) The time history of the stress is different from one of an undamped model. The maximum value of this model is smaller than one of an undamped model. Furthermore, at the 'Active' region, the response with the high frequency is eliminated and then the stress changes smoothly due to the



damping effect.

Consequently, the difference between the undamped and damped model is caused with the influence of the damping effect on the vibration mode of the eigen period T_k.

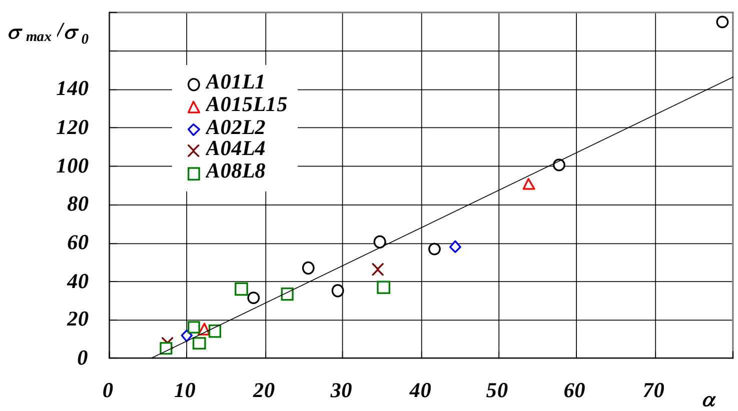


Fig.7 *Relation between the maximum stress response and parameter* α

Fig.8(a) *Displacement Time History* **Fig.8(b)** *Axial Stress Time History*

Fig.9(a) *Maximum Displacement Response* **Fig.9(b)** *Maximum Stress Response*

Fig.9(c) *Maximum Displacement Response* **Fig.9(d)** *Maximum Stress Response*

Similar to the undamped model, the maximum displacement response and stress response is shown in **Figs. 9** when changing T_k and T_p. Generally the influence of each period T_k and T_p in the case of a damped model, is similar to one of an undamped model.

Assuming the stress like an impact at the moment of tightening from loosening depends on the change of the momentum as described in the section of undamped model, parameter α should be effective in this case too. **Fig.10** shows the relation between the maximum stress response and parameter a using the results in **Figs.9**. They have a simple proportional relation. And the correlation in this case is stronger than one of an undamped model though the slope of the line is different from each other due to the damping effect.

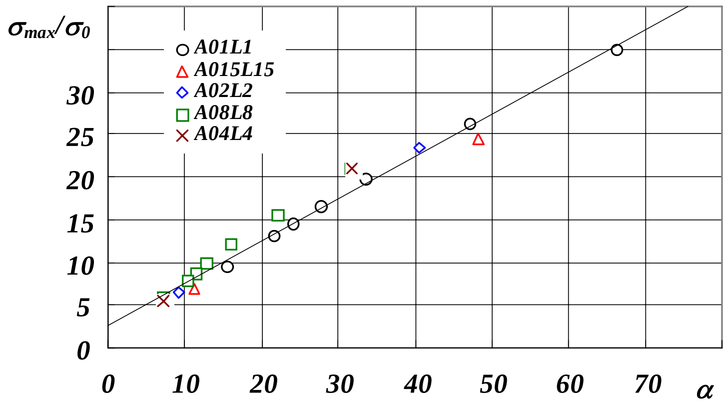
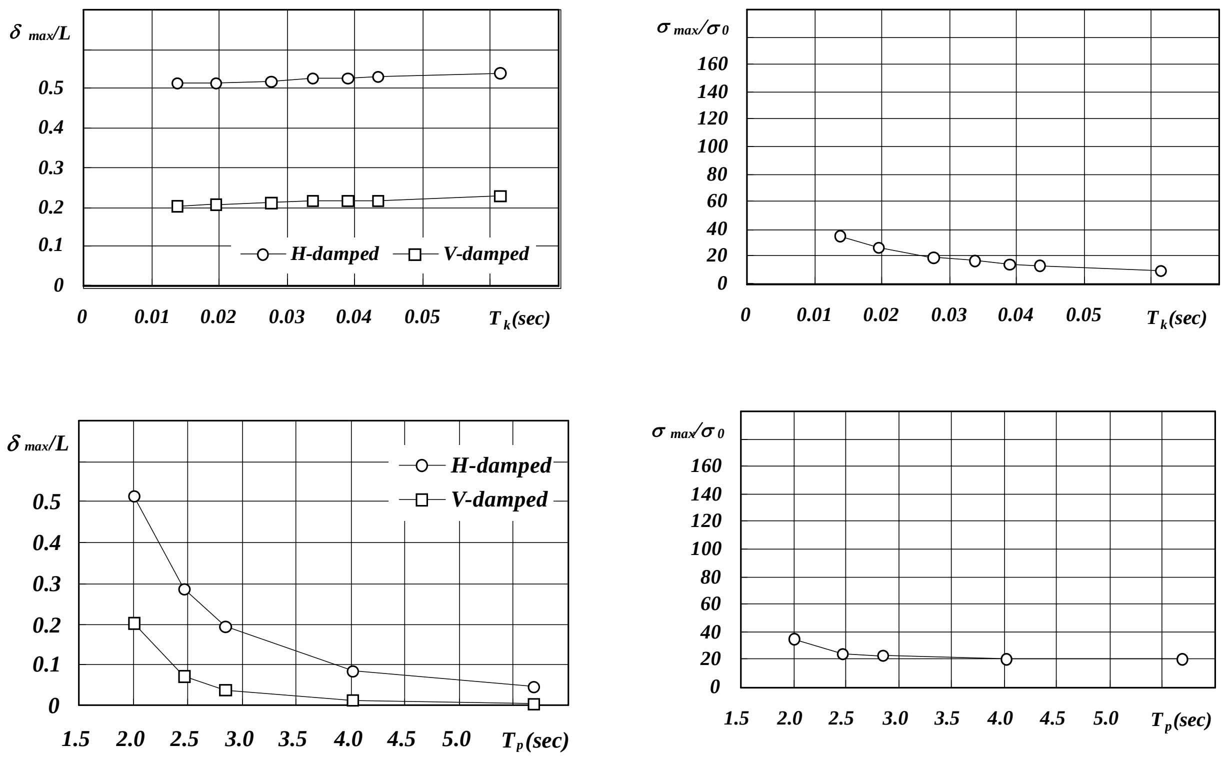


Fig.10 *Relation between the maximum stress response and parameter α for Damped Model*

Consequently our parameter is quite effective to predict the maximum stress response in the case with considering damping effect or not from these results.



Conclusion

We clarified the mechanical properties of a ‘Pendulum’ structure in cyclic loads via numerical results. Particularly, the new parameter which consists of the period T_k and T_p is quite effective and useful to predict the maximum stress response.
In our numerical method the global geometrical nonlinearity caused by large displacement of a mass node and the material nonlinearity to evaluate the loosening of cables are taken account. The consideration by such a numerical method is done in Ref.[1] to clarify the mechanical property of suspension bridge. The combination of the geometrical nonlinearity and the loosening may occur in other problems and may produce any serious damages. I guess that reports of this type research have apparently not been published to date. This type of research has just been started and is very important to clarify the true mechanical property of structures with any slender members

Reference

[1] K.T. Chau, Coupling between torsional and vertical modes of wind-induced nonlinear vibrations in Suspension Bridges due to Suspender-Loosening. Proceedings of International Conference on Advances in Steel Structures, Edited by S.L.Chan and J.G. Teng, Vol.1, 1996, pp.523-528