A Life Technique for Configuration of Roof Structures supported by Tree-like Branching Columns

Itrau MUTOH[1] and Shiro KATO[2]
1 *Dept. of Architecture & Civil Engineering, Gifu National College of Technology, Shinsei, Motosu, Gifu 501-0495, JAPAN*
Phone +81+58 320 1419, Fax +81+58 320 1429, E-mail mutoh@gifu-nct.ac.jp
2 *Dept. of Architecture & Civil Engineering, Toyohashi University of Technology, Tempaku, Toyohashi, Aichi 441-8580, JAPAN*
Phone +81+532 44 6846, Fax +81+532 44 6831, E-mail kato@tutrp.tut.ac.jp

INTRODUCTION

Engineering-based structures in contrast to architectural designed structures have been often aesthetically evaluated through interactive design/analysis on computers, e.g. Robbin 1996. Among engineers, "structural morphology" engineers are growing to interact multifarious researchers in order to create exotic geometric configurations, Wester 1994.

Design of spatial models on the basis of morphological principles can be as tools for the generation and reading, implicating of spatial forms, even useful to design, education and research, Cattan and Reissig 1997. For examples, (1)architectural design of house plans has been studied on the basis of a growth model ; with the aid of cellular automaton (CA) and genetic algorithm (GA), Rosenman 1996. And (2)structural configuration and visualization has been carried out based on the developmental model ; with the aid of L-system and GA, Mutoh and Kato 1996. Also (3)attempt to implement the AI-related database for lightweight structures in views of the conceptual structural design aid applicable to both educational program and construction engineers and designers, Sedlak 1997.

In this paper, which may be updated the previous study, the emphasis is put on the development of roof structure over a specific supporting element: tree-like branching columns. Previous study presented the applicability of ALife technique, Langton 1989, for spatial configuration to tree-like branching columns, and discussed that the drawing rules, which is converted from topological description in L-system' rules of rewriting strings, are necessary to visualize the geometric configuration. At least the outcome in visual or virtual objects seems to be like the physical "thread" models, Kolodziejczyk 1997. However, keeping the topological rules for biological background of natural plants/trees, the candidates for genes in GA associated with rewriting rules are restricted. Meanwhile only additional genes corresponding to geometric data: (1)length of branch, (2)angles of bifurcation from branch nodes and (3)contraction ratio of length over diameter as well as degree of recursions, or level of branching. Anyway, once the tree-like branching columns subjected to algorithm for canopy growth is created, the tips/apex or crown of each branches become a kind of supports to roofing frameworks.

Roofing frameworks interact with furnishing materials and/or cladding systems. The lattice framework with cladding panels of metal or glass, membranes or else, Eekhout 1993, membranes with cable-net and concrete shell roofs are of particular examples, Robbin 1996. Thus the form of roof-coverage under the condition for supporting structure configuration which is determined by means of algorithmic generation in computers can be useful for engineers and designers. Because the process documentation stored in computers through algorithmic procedures is able to access by everyone relating to architectural, structural and constructional fields.

L-SYSTEM : Overview

L-system is one of legal algorithm for analysis of natural plants/trees' branchings and growth of cells in organs, described by Lindenmayer 1968. Often called "parallel dynamic" cellular automata(CA) or "parallel" rewriting rule grammar extended Chomsky's formal language theory, L-system of the basic category is the deterministic zero-interaction L-system (D0L-system) which showed the topological form/description of the branching patterns of primitive filamentous organisms. The branching rules are though simple, able to represent

(1)dichotomous and (2)monopodial branching patters; topology structures.

The original D0L-system is defined as a triplet K=<G, w, P>, where G is a set of symbols, w is a starting symbol (axiom) and P is a set of production rules or rewriting rules. Followings illustrate how it describes branching patterns according to D0L-systems for (1) and (2) but additional notion for branches‘ bifurcation operators, brackets “[“ and “]”. Details can be found in Mutoh and Kato 1996.

A tree production (rewriting) rules replaces an edge (inter-node) in a way of which the starting node of parent, trunk, is identified with offspring‘s base, branch node, and the ending node, tip, is identified with offspring’s top. Here the most simple but basic branching topology is illustrated:

Monopodial branching: L-system definition $K_m=<G_m, w_m, P_m>$

$G_m=\{0,1,[,]\}$, w_m=“0” with $P_m=\{0 \rightarrow 11[0]0,\ 1 \rightarrow 1,\ [\rightarrow [,\ ] \rightarrow]\}$

Iteration sequence=0 to 2:

0: 0

1: 11[0]0

2: 11[11[0]0]11[0]0

Dichotomous branching: L-system definition $K_d=<G_d, w_d, P_d>$

$G_d=\{0, 1, [,]\}$, w_d=“0” with $P_d=\{0 \rightarrow 11[0][0],\ 1 \rightarrow 1,\ [\rightarrow [,\ ] \rightarrow]\}$

Iteration sequence=0 to 2

0: 0

1: 11[0][0]

2: 11[11[0][0]][11[0][0]]

in which arrow $\rightarrow$ is production operator, each intermediate level of rewriting strings, “0” and “1” mean the trunk and/or branch with uniform and homogeneous material and size with strings “[“ and “]” having arbitrary branch angle and direction.

However if the “character” of each element branch is different in-between them, following rewriting is provided. Fig.1 shows the development growth of *C.roseum*(algae), Lindenmayer 1968, to 14^{th} level of growth in case of its base of two cells denoted by D. Various type of cell development can be identified by the alphabets; from A to H. In this case, only the filamentous cell of the alphabet H can have one branch in monopodial pattern. Also as shown in Fig.2 of *C.roseum*, but by 15^{th} level of growth with the numbers; from 1 to 9. Here The main filamentous cell has its base three cells ; the number 2, without branchings. In this case, the cells with the number 9 can only bifurcate with each one branch like monopodial branching.

DRAWING RULES : Interpretation of Strings as Geometric Object

The L-system does not support drawing rules by itself. In order to give shapes (geometric description) to the generated strings, some rules for branching angles and orientations as well as diameter and length or each coordinates of growing tips is necessary. The following drawing rules are applied to visualize shapes of objects by interpreting a set of generated topological patterns.

Regular Configuration Shape

(1)Each number and/or alphabet of the generated string denotes a filamentous cell, branch which is described as a line segment or solid. A pair of brackets [left and right] in string identifies a branch node as well as specifies the direction of the branch, orientation of growing branch.

(2)The first alphanumeric character is drawn straight upward.

(3)Different alphanumeric characters have different or same cell lengths.

(4)A left bracket specifies the beginning of the branch, base, and a right bracket is identified as its end, tip.

(5)For the string 1[0]0[1] branches 0 and 1 are drawn on opposite sides of the plane. The left side is drawn first. Each branch has a constant or variable angle to the line segments 1 and 0.

(6)For the string 1[0][0] branches 0 and 0 are drawn on opposite sides of the plane with a constant or variable angle of bifurcation.

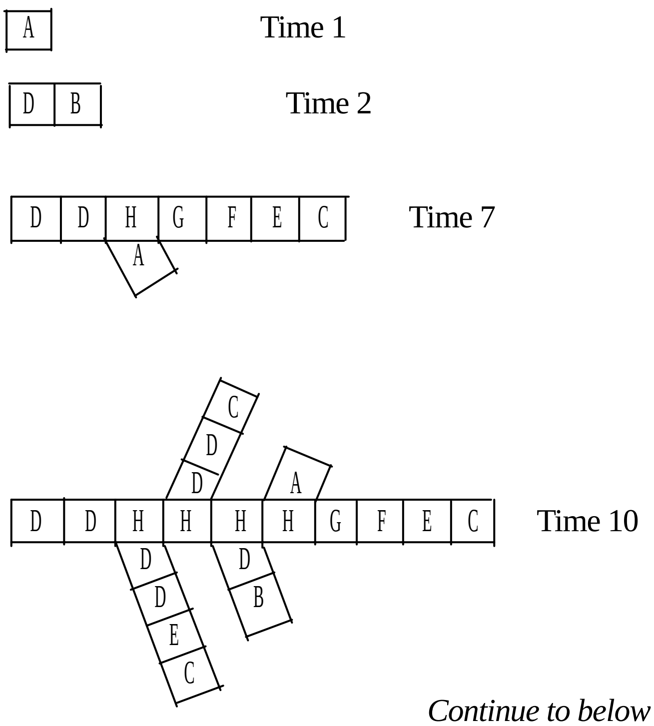
(7)Along the drawing rules, the geometric model for visualization is based on the homogeneous coordinate transformations; (a)Location/Position modified by translation, (b)Orientation/Direction modified by rotation and (c)Size/Length modified by scaling.

Table 1 Process of Filament Branching with Alphabet

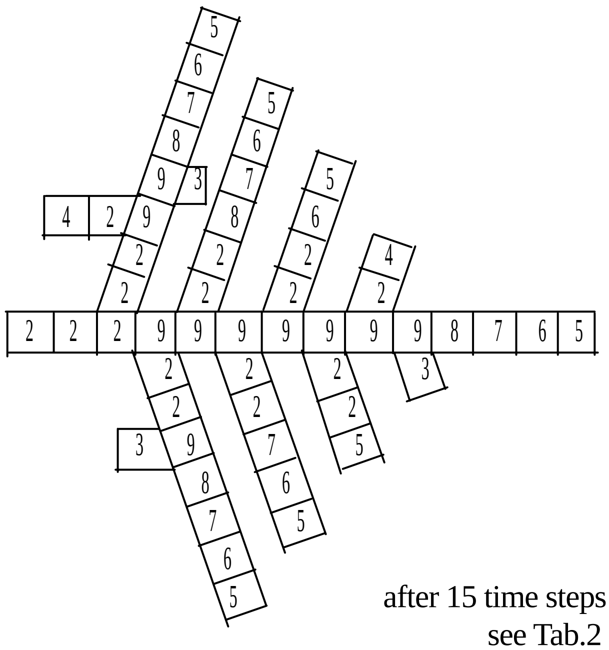
Time	Filament
1	A
2	DB
3	DDC
4	DDEC
5	DDFEC
6	DDGFEC
7	DDH[A]GFEC
8	DDH[DB]H[A]GFEC
9	DDH[DDC]H[DB]H[A]GFEC
10	DDH[DDEC]H[DDC]H[DB]H[A]GFEC
11	DDH[DDFEC]H[DDEC]H[DDC]H[DB]H[A]GFEC
12	DDH[DDGFEC]H[DDFEC]H[DDEC]H[DDC]H[DB]H[A]GFEC
13	DDH[DDH[A]GFEC]H[DDGFEC]H[DDFEC]H[DDEC]H[DDC]H[DB]H[A]GFEC
14	DDH[DDH[DB]H[A]GFEC]H[DDH[A]GFEC]H[DDGFEC]H[DDFEC]H[DDEC]H[DDC]H[DB]H[A]GFEC

Table 2 Process of Filament Branching with Numbers

Time	Filament
1	1
2	23
3	224
4	2225
5	22265
6	222765
7	2228765
8	2229[3]8765
9	2229[24]9[3]8765
10	2229[225]9[24]9[3]8765
11	2229[2265]9[225]9[24]9[3]8765
12	2229[22765]9[2265]9[225]9[24]9[3]8765
13	2229[228765]9[22765]9[2265]9[225]9[24]9[3]8765
14	2229[229[3]8765]9[228765]9[22765]9[2265]9[225]9[24]9[3]8765
15	2229[229[24]9[3]8765][229[3]8765]9[228765]9[22765]9[2265]9[225]9[24]9[3]8765

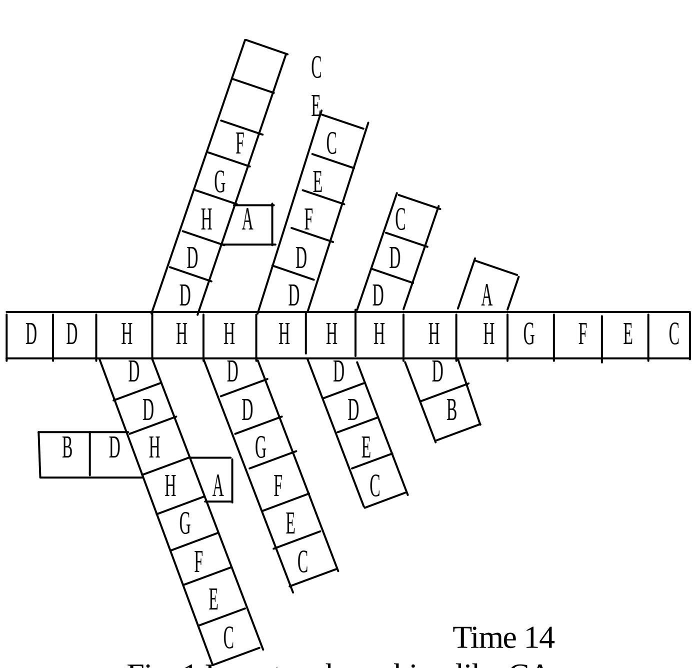


Continue to below



after 15 time steps
see Tab.2

Fig. 2 L-system branching like CA



Time 14

Fig. 1 L-system branching like CA

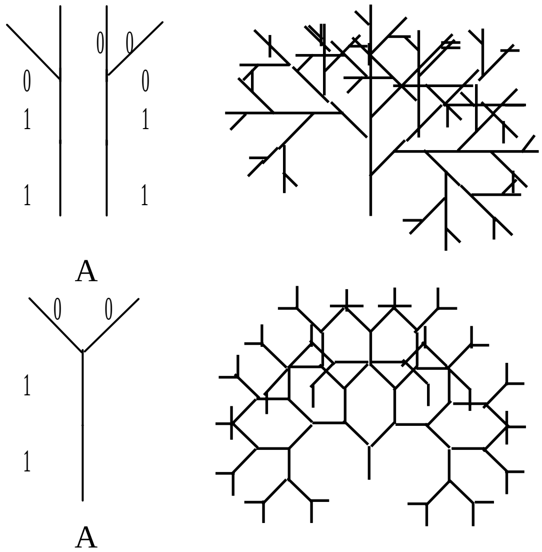


Fig. 3 Drawing rule(example)

Examples of monopodial and dichotomous branchings for visualization on the basis of the algorithm above are depicted as in Fig.3, with bifurcation angle of constant 45degrees at the level 6 of growth. Here in a 2D following algorithm for drawing is employed with the arrow for substitution operator:

1: A⟵A*T(a_I); I=I+1
0: A⟵A*T(a_I); I=I+1
[: A⟵A*R(-γ) if odd(I)
[: A⟵A*R(+γ) if even(I)
then stack(I): store its state of tip
]: pop(I): update location of tip
then A⟵A(I); and repeat. (1)

in which A means the state of position of tip, a_I =<x,y,1>, and ranslation matrix T and rotation matrix R are given below.

$$T=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx & Ty & 1 \end{bmatrix}, R=\begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Alternative can be found in usual geometric model for a 3D visualization which determine directly the geometry of object reprented its tip position (x_i,y_i,z_i) from base position(x_B,y_B,z_B) , Aono and Kunii 1984:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} + R_i * \begin{bmatrix} u*\cos\gamma i - S*L*v*\sin\gamma i \\ v*\cos\gamma i + S*L*u*\sin\gamma i \\ w*\cos\gamma i \end{bmatrix} \quad (3)$$

where u=x_B-x_A, v=y_B-y_A and w=z_B-z_A with A of base of parent and B of tip of parent; S=1/(u*u+v*v)$^{1/2}$ and L=(u*u+v*v+w*w)$^{1/2}$ of length in-between base and tip, and R_i means a contraction ratio. So if γ_I and γ_{I+1} are almost equal to zero, the branching pattern tends to monopodial branches.

Original position P=<Px, Py, Pz> is moved by translation of a vector T=<Tx, Ty, Tz>. Then rotation of angle

about each axis operates the translated position P‘ by R=<Rx, Ry, Rz>. Finally, to scale dimensions in each coordinate direction scaling by a factor S=<Sx, Sy, Sz>. However with these coordinates, each point P is defined as P=<Px, Py, Pz, 1> in a 4D space. Namely, each point <x, y, z, w> in the 4D space represents the point <x/w, y/w, z/w> in the 3D space. Here the primitive transformations above may be described by 4x4 matrices, Thalmann and Thalmann 1987.
For the examples of roofing frameworks under the constrains of regular tree-like branching support structures, Figs. 4 and 5 are depicted. Fig. 4 shows the roof like arcade over square plan-form of 4 trees. Fig. 5 shows the braced barrel vault with square mesh pattern over the trees arranged serial lines. Both the framing members of roofs are connected to the node of tips of canopy growth.

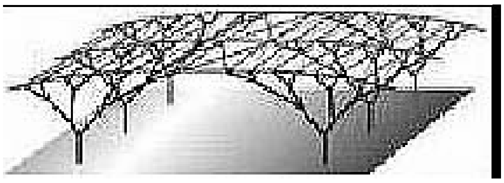


Fig.4 Arcade over trees

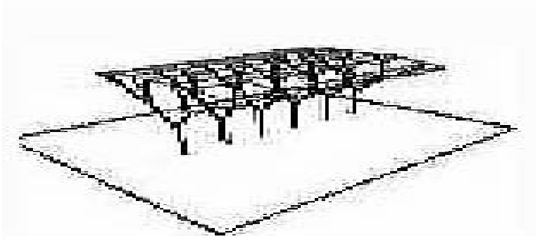


Fig. 5 Braced barrel vault

Irregular Configuration Shape

Keeping the rewriting rules constant or two type of branching topology, the drawing rules are variable: the attribute or geometric parameters assumed as each gene of chromosome for geometric objects, or phenotype. As shown in Fig. 6, one of chromosome consists of eight genes. Each gene corresponds to both the entities and values assigned, as listed in Tab. 3. In particular, the number 1 of genes which has entities of type of branchings: 1(dichotomous branching) and /or 2(monopodial branching), calls the rewriting rules and generates strings of alphanumeric characters depending on the number 8 of gene; number of branchings. Then the branch growing according to rewriting rules refers to the values in table randomly, or the data-set as gene pool which is generated randomly in the combination of values for each entities.

Chromosome | Gene1 | Gene2 | Gene3 | Gene4 | Gene5 | Gene6 | Gene7 | Gene8 |

↑ ← call

Rewritig Rules

Fig.6 Schematic Chromosome

Table 3 Definition of gene parameters

No. of Gene	Entities	Values assinged
1	Type of Branchings	1(dichotomous),2(monopodial)
2	Scale	1.1,1.2,1.3,1.4
3	Angle of branch	20,30,40,50,60,70(degs)
4	Length deviation/right	0.8,0.9,1.0,1.1
5	Length deviation/left	0.9,1.0,1.1
6	Angle deviation/right	0.7,1.0,1.3(degs)
7	Angle deviation/left	0.5,1.0,1.5(degs)
8	No.of branches	5,6,7,8,9

In this case, it is assumed that the genetic operator, cross-over and mutation without fitness evaluations:
(a)Each of six phenotype reproduced from selected chromosome.
(b)According to the probability of 1/6, 1 out of 6 selected objects is deleted.
(c)The object related to some chromosome again is selected randomly, then two particular phenotypes are determined by means of artificial selection, or aesthetically.
(d)Two phenotype out of 6 generated may be crossed over each other by three times as well as change of number of branchings with the probability of 1/5.

(e)Mutation as the trial for some amount deviation of angles ranged from +10 to –10degs., +30 to –30degs and/or +50degs, furthermore is applied to the phenotype much in irregular object, according to the probability of 1/5.

According to the procedure mentioned above, the evolution of irregularities from regular configuration of tree-like branching objects is illustrated as shown in Figs. 7,8,9,10,11,12,13 and 14. In each display,left to right in order,1to3 at top,4 to 6 at bottom mean the phenotype.

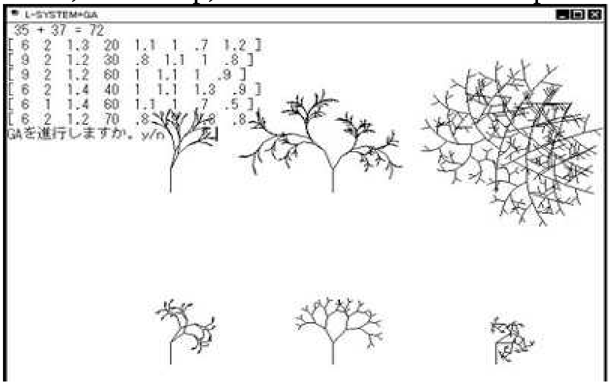


Fig. 7 6phenotypes(intial state)

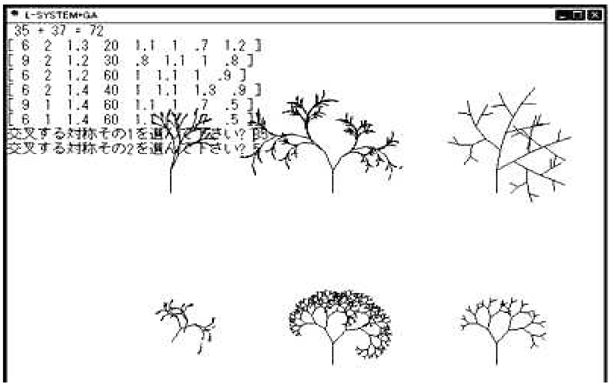


Fig.8 5th to be reproduced but 6th deleted

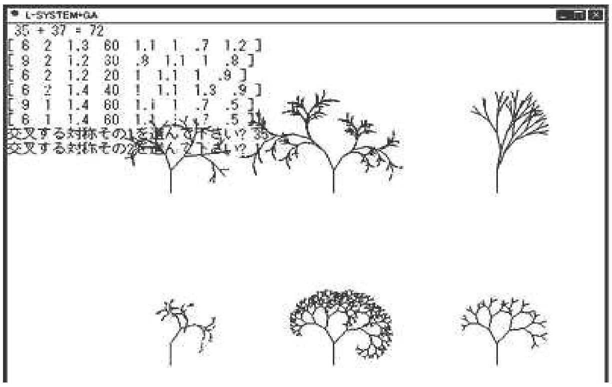


Fig.9 Reproduction by cross-over(example)

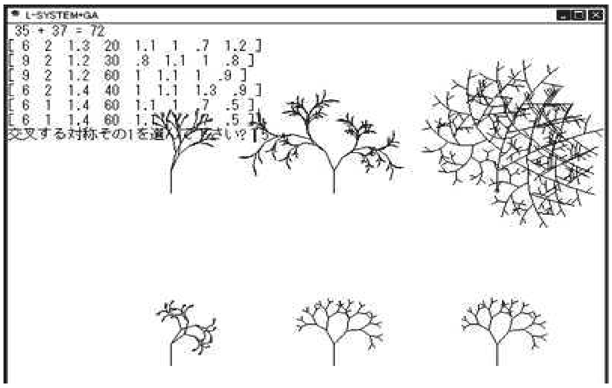


Fig.10 Selection for cross-over operation

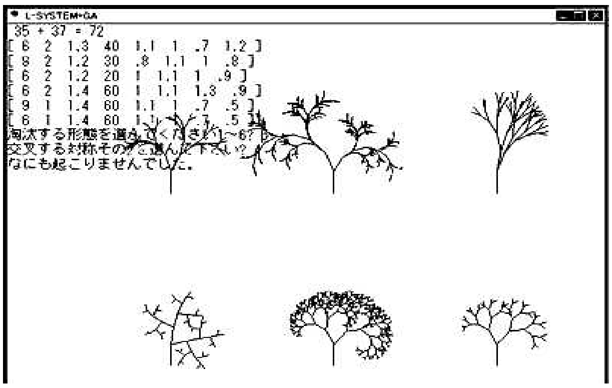


Fig.11 No development despite cross-overed

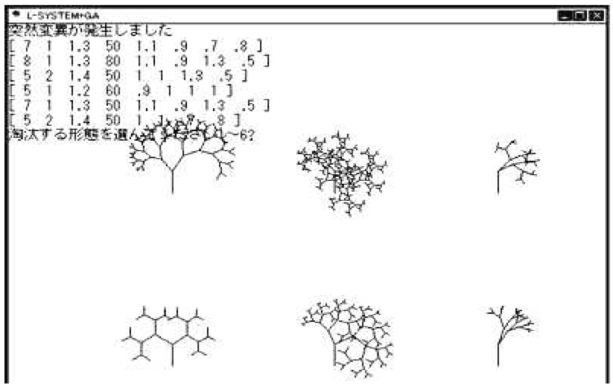


Fig.12 Multation of 2nd phenotype(example)

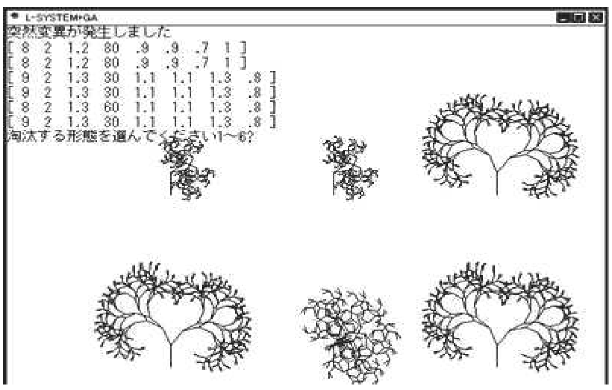


Fig.13 Selection of phenotype after mutation

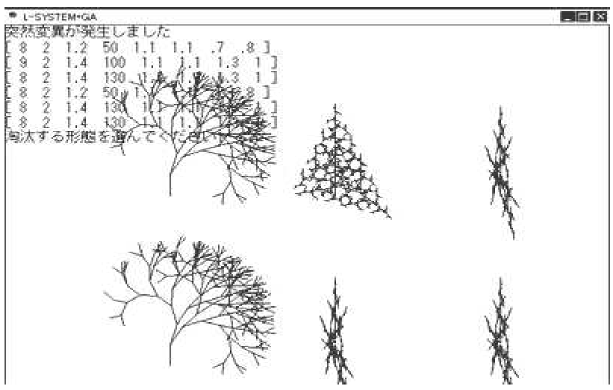


Fig.14 One of complex forms

FITNESS EVALUATION : Attempt to relate the Results form Static Analysis and Growth Parameters

From the viewpoint of the relationship between the branching structure and the mechanical behavior of a tree (natural trees), the parameters of diameter, self-weight, bending moment at each branching node, branch length and angle of branch were observed by several researchers, e.g. Thompson 1917 and Murray 1927. Then the hypothesis that the optimal/adaptable tree structures agree the minimum energy loss was examined.

The total energy loss is the sum of the energy required for the construction which is proportional to the tree volume of the branches and the energy consumption for supporting the self-weight of the branches, leaves and buds which is equal to the elastic strain energy. The outcome from comparison between observations and hypothesis, the maximum bending stress at each node of branching is constant. In so-far as this observations may be confirmed, it is evident that (1) the relationship between the diameter (periphery length) D and bending moment M acting on the branch is, M proportional to D^3, (2)the relation between the self-weight W of all the parts of the natural tree peripheral to some branching node and D is, W proportional to $D^{2.5}$ and (3)the relation between the branch length L and d is, L^2 proportional to d. Namely, the length to diameter ratio, L/D times L is constant. Resutls for a specific regular configuration are illustrated as shown in Fig. 14.Solid line means $D=0.0245*L^2$ which corresponded to a certain natural tree's branching pattern.

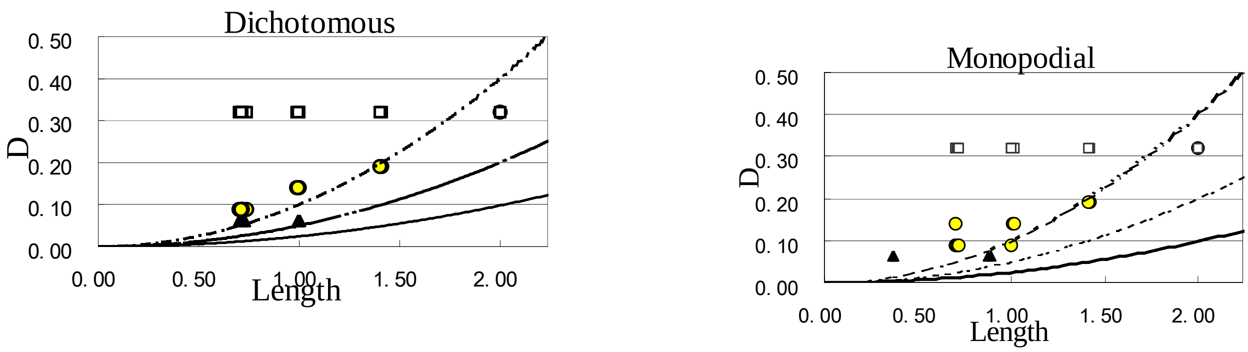


Fig. 15 Diameter and length relationship (regular configuration)

The illustration for the compatibility of growing branch size to the adaptive proportion of natural trees is investigated through the static analysis: elastic linear analysis for the tree-like branching columns. As summarized in Tab.4, the member of branching trees is 2 meter length of trunk(stem) with section sizes according to the number of branchings. The member of pipe section steel material with rigid joint connection at each node under $20kN/m^2$ of tributary area for tips of canopy. The member slenderness is ranged from about 6 to 30, then the member buckling is not considered. The analytical models for (1)dichotomous and (2)monopodial branching of regular configurations are first assumed as metioned before. However the constituent member sections are either uniform despite different number of growth level or variable dependent to growth level as listed in Tab.4. Also the tree-branching supports with or without roof frames are examined. The results for bending moment distribution and displacements are shown as in Fig. 17 and 18. The ratio, D/L, for the results with observed relation inserted is depicted as shown in Fig. 15.

Next for irregular configuration as shown in Figs. 19 and 20, the moment, axial force and displacement distributions are illustrated. Here assumed chromosome as in Fig. 6 consites of genes for geometric parameters defined as listed in Tab.3.

Also the ratio, D/L, for the results is summarized as shown in Fig. 16. The inserted curves are of empirical equation due to a kind of adaptability to constant moment at each node of branches as mentioned before.

If the growth of tree-like supports tends to be in the ratio for natural trees' branching growth which may be optimal then artificial branching operation due to GA is more adaptable in practice. However all the members are different each other as well as connected with different angles. Also, each phenotype generated by GA operation is measured its fitness due to the ratio of D/L for natural trees. Examples for roof framing system based on both regular and irregular branching supports are illustrated.

Table 4 Member Sizes

No.	Diameter * Thickness
1	318.5 * 6.9(mm)
2	190.7 * 5.3(mm)
3	139.8 * 4.0(mm)
4	89.1 * 3.2(mm)
5	60.5 * 2.3(mm)

Note: branch No.5 as Tip jointed to roof frames, branch No.1 as trunk branch. In case of same branch members by No.1.

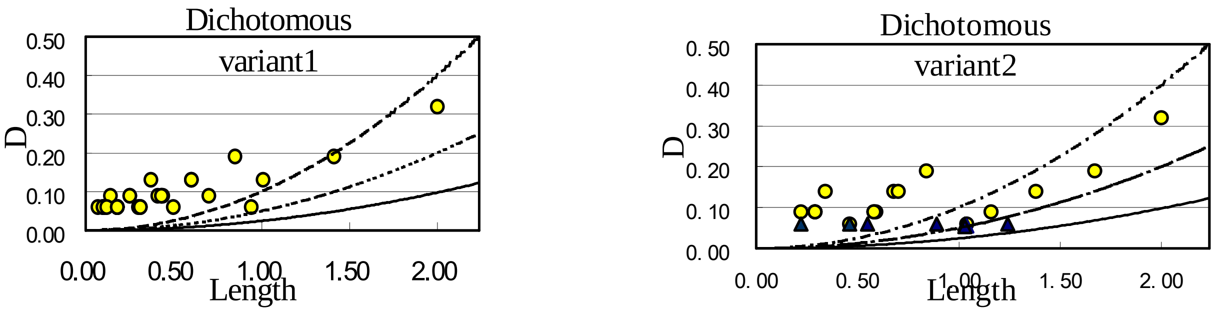
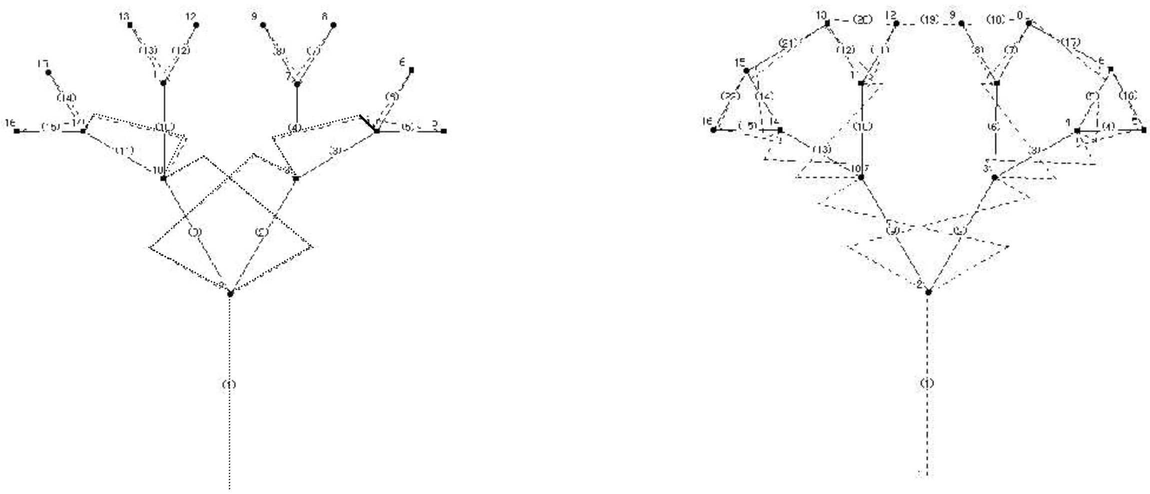
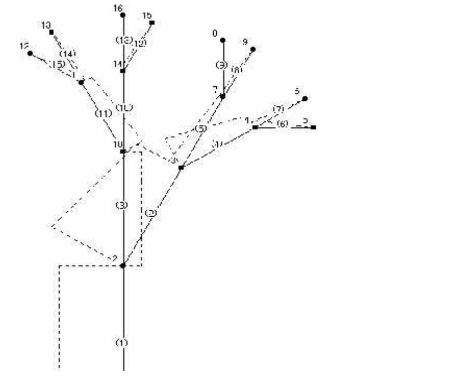


Fig. 16 Diameter and length relationships for irregular variant configurations(dichotomous branches)

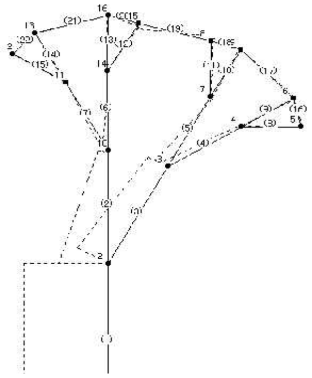


(a)Momentdistribution(Mmax=66kNm)m (b)Momentdistribution(Mmax=3.5kNm)

Fig.17 Dichotomous branchings with / without roof frames

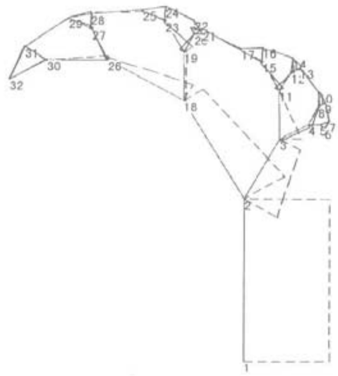


(a)M_{max}=85kNm



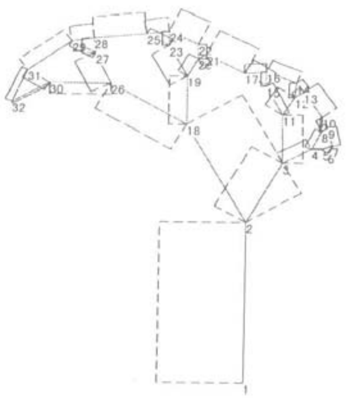
(b)M_{max}=19.9kNm

Fig.18 Monopodial branching with / without roof frames



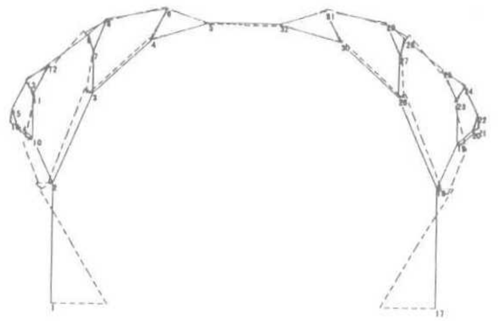
M_{max}=83.8kNm

Max.vertical displacement=100m

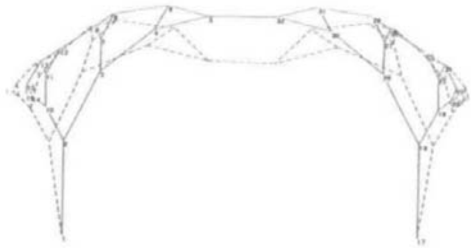


Max.axial force=188kN

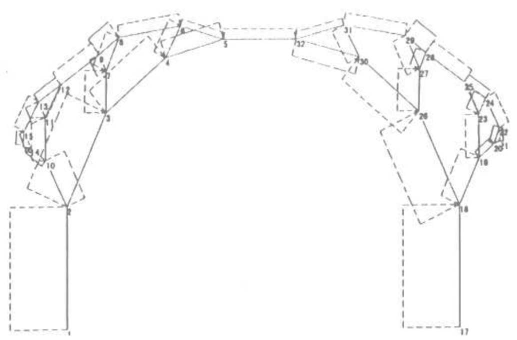
Fig.19 Irregular configuration with roof frames



M_{max}=24.4kNm



Max.vertical displacement=6mm



Max axial force=94.3kN

Fig.20 Arcade with Duplication of left figure

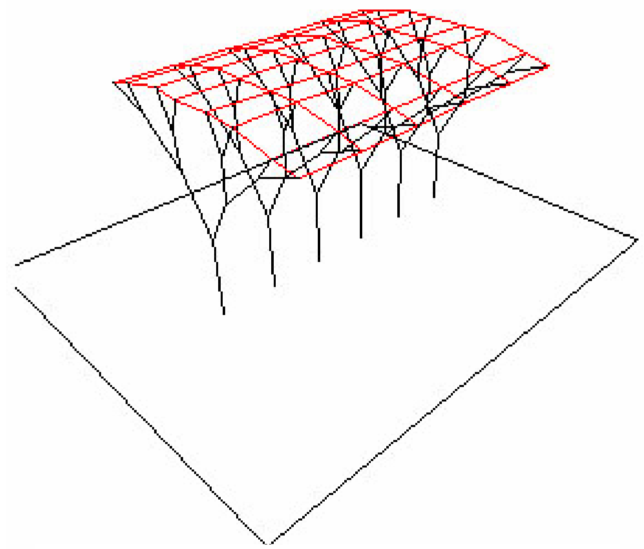


Fig.21 Regular braced barrel vault

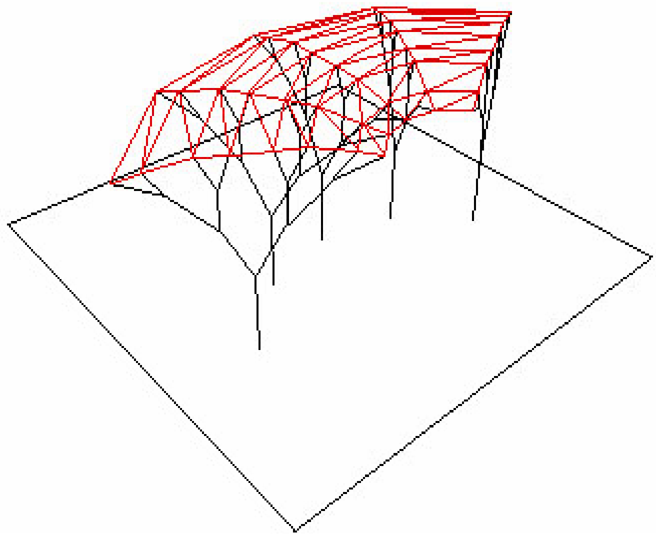


Fig.22 Irregular curved roofing

For roof framing system which is defined due to a canopy of branching tree-like supports, Fig21 means barrel grid vault over regular form. In Fig.22 position of tree trunks is arranged arbitray in fan-like form. The further study into such an arrangement rules can be developed by means of cellular automaton(CA) in conjuction with GA applied to L-system's rewriting rules directly.

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