Elasto-Plastic Buckling Behaviour of Rigidly Jointed Single-Layer Latticed Domes

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SUMMARY

The purpose of this paper is to estimate the elasto-plastic buckling load of rigidly jointed single-layer latticed domes under uniform load. The value of λs is adopted as a slenderness ratio of a dome. To examine the fundamental elasto-plastic buckling behaviour of the dome without geometrical initial imperfections, the elasto-plastic analysis is carried out. Next, the buckling behaviour of the domes with geometrical initial imperfections is examined. As a result, the elasto-plastic buckling load of the domes is estimated by using Modified Dunkerley Formulation as a function of λs.

1. INTRODUCTION

The single-layer latticed dome is a framed structure, and shows a similar behaviour with continuous shells. It is made clear by experimental results[1)] that the material nonlinearity influences the buckling load of the single-layer latticed dome. In recent years, the elasto-plastic buckling analysis is carried out with the progress of nonlinear analysis method.

The purpose of this paper is to estimate the elasto-plastic buckling behaviour of rigidly jointed single-layer latticed domes under uniform load. To begin with, the linear buckling analysis, the elastic buckling analysis and the elasto-plastic buckling analysis are carried out for domes without geometrical initial imperfections. The elasto-plastic buckling behaviour of rigidly jointed single-layer latticed dome is made clear by comparing these results. Next, we discuss the effect of a geometrical initial imperfection on the buckling load reduction

As a result, the elasto-plastic buckling load of the dome is estimated by using Modified Dunkerley Formulation as a function of λs[2)].

2. ELASTO-PLASTIC BUCKLING BEHAVIOR OF RIGIDLY JOINTED SINGLE-LAYER LATTICED DOMES WITHOUT GEOMETRICAL INITIAL IMPERFECTIONS

2.1 Method of Analysis

Figure 1 shows the geometry of rigidly jointed single-layer latticed dome. For the boundary condition, the nodes of the outer ring are pin supported. The uniform load is applied vertically at each of free joints. Dimensions of domes are shown in Table 1, and the member properties of the tubular section are shown in Table 2. Young's modulus E=2.06Å~10^5 MPa, the yield stress σy=235.4 or 294.2 MPa and modulus of strain hardening Et=E / 100. The stress-strain relationship is assumed to be bi-linear.

Three methods of analysis are used here to determine the buckling load. These methods are the linear eigenvalue analysis, the elastic buckling analysis and the elasto-plastic buckling analysis. Taking advantage of symmetry, a half of the dome is analyzed.

2.2 Slenderness ratio of a dome

The value of λs(Eq.(1)) is adopted as a slenderness ratio of a dome.

$$\lambda s = \sqrt{\sigma y / \sigma_{cl}}$$
$$= \sqrt[4]{3(1-\upsilon^2)} \sqrt{\frac{\sigma y}{E}} \sqrt{\frac{R}{te}}$$

where σ_{cl} is the classical buckling stress of a spherical shell, Poisson's ratio υ =0.3 and the effective shell thickness $te = 2\sqrt{3}\,i$. The values of λs are shown in Table 3.

2.3 The elasto-plastic buckling behaviour of rigidly jointed single-layer latticed domes without geometrical initial imperfections

The load-displacement relationships of the domes are shown in Figure 3. The load P is normalized by the elastic buckling load $P_{cr}^{el}{}_{(p)}$, the displacement D is normalized by the displacement $D_{cr}^{el}{}_{(p)}$ at the elastic buckling load. The open triangle Å§ gives the member initial yield load $P_y^{pl}{}_{(p)}$, and the closed triangle Å•gives the buckling load. For the dome with λs=0.63, the ratio of the elasto-plastic buckling load to the elastic buckling load($P_{cr}^{pl}{}_{(p)}$ / $P_{cr}^{el}{}_{(p)}$) is 0.64, and the elasto-plastic buckling load is 1.23 times larger than the member initial yield load. On the other hand, for the dome with λs=0.97, the ratio of the elasto-plastic buckling load to the elastic buckling load approaches unity, and $P_{cr}^{pl}{}_{(p)}/P_y^{pl}{}_{(p)}$=1.02.

Figure 4 shows the collapse mechanism. The thick solid line gives the yield members. For the dome with λs=0.63, central members yield at first. And yield member gradually propagates over all. Finally, all members except for the first and second ring become yield. For the dome with λs=0.80, members will yield in the same way of the dome with λs=0.63, but central members do not yield. For the dome with λs=0.97, corner members yield at first, and second ring yield. This behaviour is different from the other models.

The deformations at the elasto-plastic buckling load are shown in Figure 5. The dome with λs=0.63 is collapsed by general buckling. On the other hand, the dome with λs=0.97 is collapsed by node buckling.

The relation between equivalent buckling wave length *l*eq and $\xi = 12\sqrt{2} / (\lambda \cdot \theta)$3) is shown in Figure 6. This ξ is the geometrical parameter that represents the shell-likeness of the dome. The *l*eq is normalized by member length *l*. Figure 6(a) shows the results obtained by linear buckling analysis, and *l*eq in this figure is calculated by Eq.(2) and (3). On the other hand, Figure 6(b) shows the results obtained by the elasto-plastic buckling analysis, and *l*eq in this figure is calculated by Eq.(2). Here, Ncr(p) in Eq.(2) is a maximum axial member force at the elasto-plastic buckling load.

$$N_{cr(p)} = \pi^2 EA / \left(\frac{leq}{i}\right)^2$$
$$N_{cr(p)} = N_{i(p)} \cdot P^{lin}_{cr(p)}$$

here, $N_{i(p)}$ is a maximum axial member force under unit uniform load and $P_{cr}^{lin}{}_{(p)}$ is the linear buckling load.

For the linear buckling analysis, the buckling wave length depends on ξ, and all of the domes with ξ<5 become $l_{eq}^{lin}{}_{(p)}/\ l$ <1.0. This means that these domes are collapsed by member buckling. For the elasto-plastic analysis, the buckling wave length depends on λ rather than ξ. Namely, the buckling wave length is determined by λ. Moreover, the buckling wave length is longer than that of the linear buckling analysis.

Compared with the elastic buckling behavior, some characteristics of the elasto-plastic buckling behaviour are shown as follows: As the dome which the value of λs is small, the elasto-plastic buckling load is smaller than the elastic buckling load, because a lot of members yield. These domes are collapsed by general buckling. As the value of λs increases, number of yield member decrease and the ratio of the elasto-plastic buckling load to the elastic buckling load approaches unity. These domes are collapsed by node buckling.

2.4 The estimation of elasto-plastic buckling load of the rigidly jointed single-layer latticed domes without geometrical initial imperfections

We express the elasto-plastic buckling load in terms of generalized slenderness ratio Λ[3] (See Figure 7). The generalized slenderness ratio Λ is defined as a square root of the quantity obtained through dividing Ny(=σy•A) by the linear buckling axial force $N_{cr}{}^{lin}{}_{(p)}$. The results are estimated by Modified Dunkerley Formulation[4]. In other words, the elasto-plastic buckling load of rigidly jointed single-layer latticed dome is estimated by member axial force.

Next, we express the elasto-plastic buckling load in terms of member yield load Py using the value of λs (See Figure 8). Here, Py is calculated by ;

$$Py = 6\sigma y \cdot A \cdot \theta$$

The results almost coincide with Modified Dunkerley Formulation expressed in Eq.(5).

$$\frac{1}{\gamma/\lambda s^2} \cdot \left[\frac{P^{pl}_{cr(p)}}{Py}\right] + \left[\frac{P^{pl}_{cr(p)}}{Py}\right]^2 = 1$$

here, knock-down factor γ=1.0.

The elasto-plastic buckling load is estimated by both estimations.

3. The buckling behavior of rigidly jointed single-layer latticed domes with geometrical initial imperfections

3.1 Method of analysis and assumption of geometrical initial imperfection

The analytical model and the method of analysis are the same ways as foregoing chapter. In this chapter, the buckling load reduction influenced by geometrical initial imperfections is discussed. The geometrical initial imperfections are assumed as follows.

(i) a maximum amplitude node of the first mode obtained from linear eigenvalue analysis

(ii) a maximum displacement node at the elasto-plastic buckling load without geometrical initial imperfections

(iii) similar shape of the first mode obtained from linear eigenvalue analysis

(iv) similar shape of deformation at the elasto-plastic buckling load of the dome without geometrical initial imperfection

In each case, the maximum amplitude of the imperfection is assumed to be 0.2*te*. The initial imperfection is given only in the vertical direction at each node.

The examples of geometrical initial imperfection rules (i) and (iii) are shown in Figure 9. Table 4 shows the node number that geometrical initial imperfection is assumed by rules (i) and (ii).

Figure 10 shows the ratio of the elasto-plastic buckling load of the dome with geometrical initial imperfection $P_{cr}{}^{pl}{}_{(i)}$ to the elasto-plastic buckling load of the dome without geometrical initial imperfection $P_{cr}{}^{pl}{}_{(p)}$. Figure 10(i) shows the $P_{cr}{}^{pl}{}_{(i)}$ / $P_{cr}{}^{pl}{}_{(p)}$ in the case of the dome with geometrical initial imperfection (i). Here, taking note of the results of θ=3Åã, this ratio becomes large as the value of λs increases. This reason is shown in the follows. The magnitude of node imperfection is defined by the effective shell thickness. Thus, as the value of λ increases, namely, as the value of λs increases, these domes have small imperfection at the apex of the dome. Figure 10(ii) shows the $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ in the case of the dome with geometrical initial imperfection (ii). The domes that λs is large have the imperfection at the corner node of the dome. For this reason, as the value of λs increases, $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ becomes small. Figure 10(iii) shows the $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ in the case of the dome with geometrical initial imperfection (iii). As the value of λs increases, $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ becomes small. Especially, the buckling reduction of two domes is large. Figure 10(iv) shows the $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ in the case of the dome with geometrical initial imperfection (iv). For the dome with λsÅÖ0.80, $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ Å‡0.85. However, for the dome with λs>0.80, as the value of λs increases, $P^{pl}_{cr(i)} / P^{pl}_{cr(p)}$ becomes small.

Next, we discuss the buckling behaviour of each dome. The collapse mechanism at the elasto-plastic buckling load of the dome with λs=0.80 with imperfection (i) is shown in Figure 11(a). Since this dome has the imperfection at the apex, only members connected to the apex yield. The collapse mechanism at the elasto-plastic buckling load of the dome with λs=0.97 with imperfection (ii) is shown in Figure 11(b). Since this dome has the imperfection at the node of the corner, the stress concentrated member of the dome yield at first. Therefore the elasto-plastic buckling load of the dome with imperfection is smaller than that of the dome without imperfection.

To examined the elasto-plastic buckling behaviour of the dome with imperfection (iii), the equivalent buckling wave length ratio $l^{pl}_{eq(p)} / l^{lin}_{eq(p)}$ is shown in Figure 12. As the value of λs increases, $l^{pl}_{eq(p)} / l^{lin}_{eq(p)}$ approaches unity. In other words, imperfection wave length is equal to elasto-plastic buckling wave length of the dome without imperfection. For this reason, $P^{pl}_{cr(i)}/P^{pl}_{cr(p)}$ is considered to be small.

In the case of the dome with geometrical initial imperfection (iv), imperfection wave length is equal to elasto-plastic buckling wave length of the dome without imperfection. The ratio $P^{pl}_{cr(i)}/P^{pl}_{cr(p)}$ is not very small. However, for the domes with λs=0.97, $P^{pl}_{cr(i)}/P^{pl}_{cr(p)}$ is small. This is the same behaviour as that of the dome with imperfections.

Compared with the elasto-plastic buckling behaviour of the dome without geometrical initial imperfections, some characteristics of the elasto-plastic buckling behaviour with the geometrical initial imperfection are presented as follows: For the ideal domes with λs<0.80, the elasto-plastic buckling mode is different from the elastic buckling mode. Therefore, the elasto-plastic buckling load that the dome has the imperfection at the maximum displacement node decreases suddenly. For the domes with 0.80 < λs < 1.0, the domes are collapsed by node buckling. The elasto-plastic buckling load of these domes is greatly influenced by imperfections.

3.2 The estimation of the elasto-plastic buckling load of the dome with geometrical initial imperfection

The elasto-plastic buckling load of the dome with geometrical initial imperfection is estimated by the same way of the elasto-plastic buckling load of the dome without geometrical initial imperfection.

The results are shown in Figures 13 and 14. In the estimation of Λ, the elasto-plastic buckling load of the dome with geometrical initial imperfections is estimated by Modified Dunkerley Formulation with knock-down factor γ=0.43; γ=0.43 is shown in recommendation of IASS[5]. On the other hand, in the estimation of λs, the results almost coincide Modified Dunkerley Formulation with knock-down factor γ= 0.35. In the former, the generalized slenderness ratio Λ and the maximum axial force member are determined by the result of the linear buckling analysis for the dome with imperfections. Therefore the elasto-plastic buckling load is estimated in spite of the shape of the geometrical initial imperfection. In the later, Py is calculated by the member connected to the apex of the dome, and λs is calculated by the dome without geometrical initial imperfections. Therefore, in this case the knock-down factor γ is smaller than 0.43.

4. CONCLUSION

In this study, the behaviour of the rigidly jointed single-layer latticed dome is examined by numerical analysis. Summaries of the results obtained may be as follows.

(1)The elasto-plastic buckling behaviour of the rigidly jointed single-layer latticed dome is estimated by the slenderness ratio of the dome λs.

(2)For the domes without geometrical initial imperfections, for the domes with λs < 0.80, the elasto-plastic buckling load is smaller than the elastic buckling load. And these domes are collapsed by general buckling. For the domes with 0.80 < λs < 1.0, the domes are collapsed by node buckling.

(3)For the domes with geometrical initial imperfections, as the value of λs increases, the elasto-plastic buckling load decreases. Especially, the elasto-plastic buckling load of the dome which is collapsed by node buckling is greatly influenced by geometrical initial imperfections.

(4)The elasto-plastic buckling load is estimated by using Modified Dunkerley Formulation as the function of λs as well as the function of Λ.

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