Inelastic lateral buckling strength and design of steel arches by Yong-Lin Pi and NS Trahair
Address: Department of Civil Engineering, University of Sydney, NSW, 2006, Australia
Phone: 93515217, Fax: 93513343, E-mail: linp@civil.su.oz.au

Inelastic Lateral Buckling Strength and Design of Steel Arches

YONG-LIN PI * and N. S. TRAHAIR
Department of Civil Engineering
University of Sydney, NSW, 2006, Australia

ABSTRACT

Arches resist general loading by a combination of axial compression and bending actions. Under these actions, an arch loaded in-plane may suddenly deflect laterally and twist out of the plane of loading and fail in a flexural-torsional buckling mode. This paper investigates the inelastic flexural-torsional buckling strength and design of steel arches under general loading using an advanced nonlinear inelastic finite element method of analysis.

It is found that the arch subtended angle significantly affects its flexural-torsional buckling strength. The strength decreases as the subtended angle increases. The effects of the loading distribution are important. The maximum moments of arches under a central concentrated load are generally lower than those of arches under a quarter point concentrated load, and the maximum moments of arches under a load uniformly distributed along the entire arch are generally lower than those of arches under a load uniformly distributed along a half arch.

Modifications of the design rules for steel beam-columns are developed for the design of steel arches under general loading, based on the finite element analysis results.

INTRODUCTION

Arches resist general loading (Fig. 1) by a combination of axial compression and bending actions which vary along the arch. Under these actions, an arch may suddenly deflect laterally and twist out of the plane of loading and fail in a flexural-torsional buckling mode. Studies of the inelastic flexural-torsional buckling and strength of steel arches under general loading are limited, while very few steel design standards give rules for designing arches against inelastic flexural-torsional buckling under general loading.

Guide to stability design (1988) and *Structural stability design* (1997) included the numerical results of Komatsu and Sakimoto (1977) and Sakimoto and Komatsu (1983) which indicated the effects of the rise-to-span ratio (which depends directly on the subtended angle of the arch) on the out-of-plane ultimate strength are not important and that the design rules for a column can directly be used in the determination of the ultimate strength of through-type steel arches of box section. Recently, Pi and Trahair (1998) studied the out-of-plane inelastic buckling and strengths of circular steel I-section arches either in uniform compression or in uniform bending using a nonlinear inelastic finite element method. They found that the subtended angles have significant effects on their inelastic flexural-torsional buckling and strengths. They modified design rules for columns and beams to develop a design method for arches in uniform compression and in uniform bending which allows for the effects of the subtended angle.

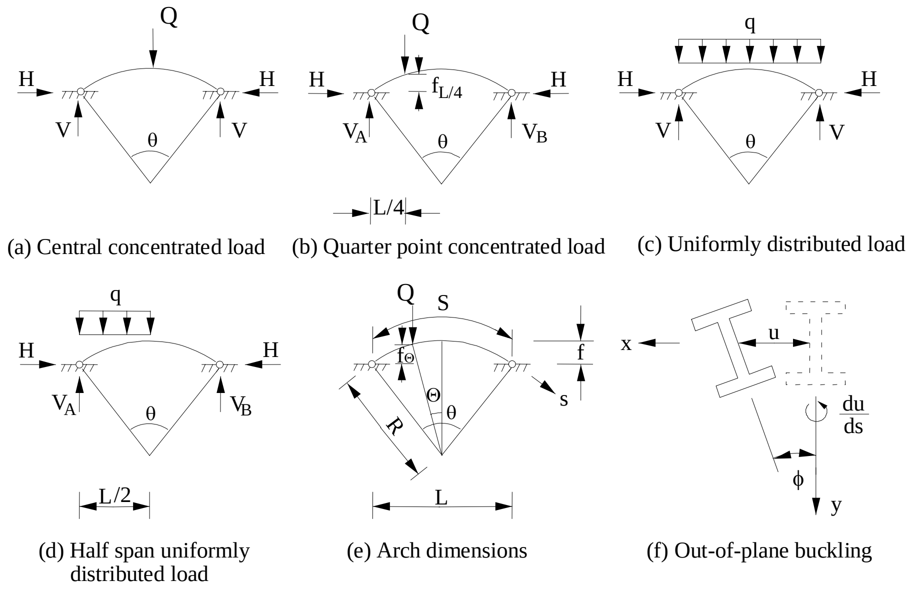


Fig. 1 Arch loading and buckling

However, arches in either uniform bending or uniform axial compression are rare. In most cases, arches are acted on by vertical loads which induce combined axial compression and bending. Preliminary suggestions for the out-of-plane design of such arches were made by Papangelis and Trahair (1993). The purposes of the present paper are to study the inelastic flexural-torsional buckling strengths of circular steel I-section arches under vertical loading and the effects of various factors on their strengths, and to develop a design method.

FINITE ELEMENT MODEL FOR ARCHES

A nonlinear inelastic finite element model for the advanced analysis of the out-of-plane behaviour of steel arches has been developed by including material inelasticity (Pi and Trahair 1994, 1995) in a nonlinear elastic finite element model (Pi and Trahair 1996a). The formulation of the model is based on the assumptions of small strains and no local and distortional buckling, and includes the effects of large deformations, the in-plane curvature, the initial crookedness and twist, and residual stresses. Numerous examples have verified that the finite element model is accurate, effective, and efficient (Pi and Trahair 1995, 1996a,b, 1998).

Full details of the model and of the method of solution of the resulting nonlinear incremental-iterative equilibrium equations are given by Pi and Trahair (1994, 1995, 1996a).

STRENGTHS OF ARCHES IN UNIFORM COMPRESSION OR UNIFORM BENDING

The finite element model was used by Pi and Trahair (1998) to study the inelastic flexural-torsional buckling strengths of arches either in uniform compression or in uniform bending. The study showed that the methods used to design columns or beams could be modified to approximate the effects of the subtended angle on the flexural-torsional buckling strength of an arch.

The nominal capacity N_{ca} of an arch in uniform compression was proposed as (Pi and Trahair, 1998)

$$N_{ca} = \alpha_{ca} N_Y \tag{1}$$

where the squash load of the cross section $N_Y = A\sigma_y$ with A = the area of the cross section and σ_y = yield stress, and α_{ca} is the arch slenderness reduction factor which can be obtained from the AS4100(1990) column rule using a modified slenderness $\lambda_{ac} = \sqrt{N_Y / N_{ya}}$ of an arch in uniform compression instead of the modified slenderness of a column, where N_{ya} is the lowest elastic flexural-torsional buckling load of an arch in uniform compression (Yang and Kuo 1987, Pi and Trahair 1998).

The variations of the dimensionless nominal capacity N_{ca} / N_Y with the modified slenderness λ_{ac} were compared with the finite element results in Pi and Trahair (1998), and good agreement was obtained.

The nominal capacity of an arch in uniform bending was proposed as (Pi and Trahair, 1998)

$$M_{abx} = \alpha_{sa} M_{px} \tag{2}$$

where M_{px} is the full plastic moment of the cross section about the major principal axis and α_{sa} is the arch slenderness reduction factor in uniform bending given by

$$\alpha_{sa} = 0.6\left(\sqrt{\left(M_{px} / M_{ysa}\right)^2 + 3} - \left(M_{px} / M_{ysa}\right)\right) \tag{3}$$

where M_{ysa} is the lowest elastic flexural-torsional buckling moment of an arch in uniform bending (Yang and Kuo 1987, Pi and Trahair 1998).

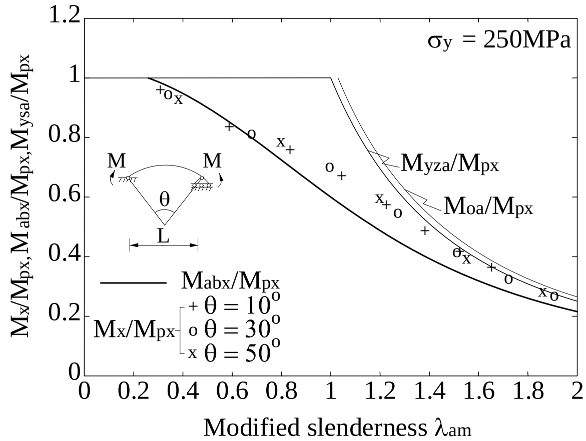


Fig. 2 Strengths of steel arches in uniform bending

The variations of the dimensionless nominal capacity M_{abx} / M_{px} with the modified slenderness $\lambda_{am} = \sqrt{M_{px} / M_{ysa}}$ are compared with the finite element results in Fig. 2, which shows that (2) provides good predictions of the strengths of arches in uniform bending.

BUCKLING AND STRENGTH

Effects of Subtended Angle

The arches used in this paper are assumed to have the cross-section, residual stresses, and stress-strain curve shown in Fig. 3 and the initial lateral crookedness u_0 and twist ϕ_0 defined by $u_0 / u_{c0} = \phi_0 / \phi_{c0} = \sin(\pi s / S)$, $u_{c0} = S / 1000$, and $\phi_{c0} = u_{c0} N_{ya} / M_{ysa}$ where u_{c0} and ϕ_{c0} are the initial central crookedness and twist, S is the developed length, and s is the coordinate along the arch axis.

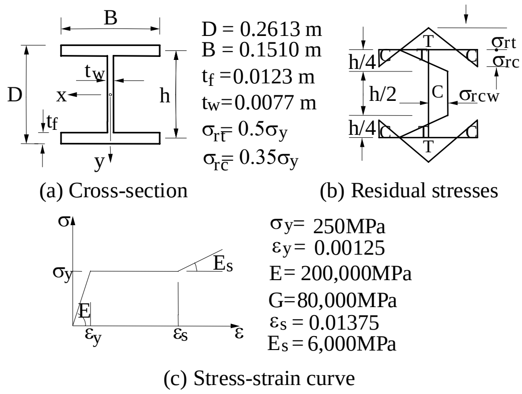


Fig. 3 Cross-section, residual stresses and stress-strain curve

The significant quantities which are likely to be used in the design of a steel arch are the maximum bending moment M_m and the maximum axial compression N_m in the arch, and in practice, designers will prefer to use a first order elastic analysis to calculate these. For this reason, the findings of this paper are presented in terms of the maximum values of the dimensionless maximum moment M_m / M_{px} and the dimensionless maximum compression N_m / N_Y

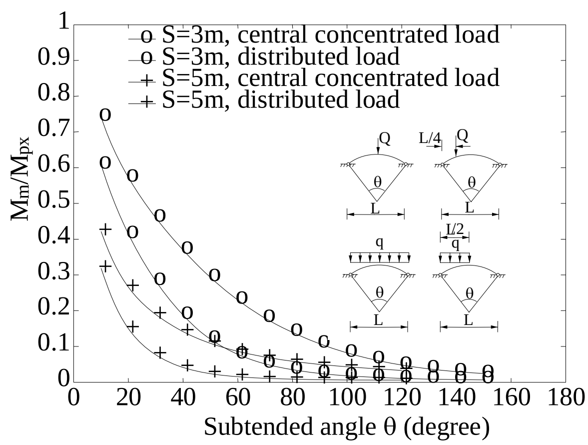


Fig. 4 Effects of subtended angle

The effects of the subtended angle on the strengths of arches are demonstrated by the variations of the dimensionless maximum moment M_m / M_{px} with the subtended angle θ in Fig. 4. For all the arches, the dimensionless maximum moment M_m / M_{px} decreases as the subtended angle θ increases. For the same subtended angle, the values of M_m / M_{px} for the arches with longer developed lengths ($S = 5$m) are lower than those for the arches with shorter developed lengths ($S = 3$ m) because the arches with longer developed lengths are more slender. It can also be seen that the maximum moments M_m of arches with large subtended angles are much lower than the full plastic moment M_{px} of the cross section because these arches are very slender.

Effects of Loading

The effects of loading on the strengths of arches are demonstrated by the variations of the dimensionless maximum moment M_m / M_{px} with the modified slenderness λ_{am} in Fig. 5. The dimensionless maximum moments M_m / M_{px} decrease as the modified slendernesses λ_{am} increase. The dimensionless maximum moments M_m / M_{px} of the arches subjected to a central concentrated load are lower than those of the arches subjected to a quarter point concentrated load. The dimensionless maximum moments M_m / M_{px} of the arches subjected to a load uniformly distributed along the horizontal projection of the entire arch are generally lower than those of the arches subjected to a load uniformly distributed along the horizontal projection of a half arch, except for a very short arch ($S = 1$ m, $\lambda_{am} = 0.33$).

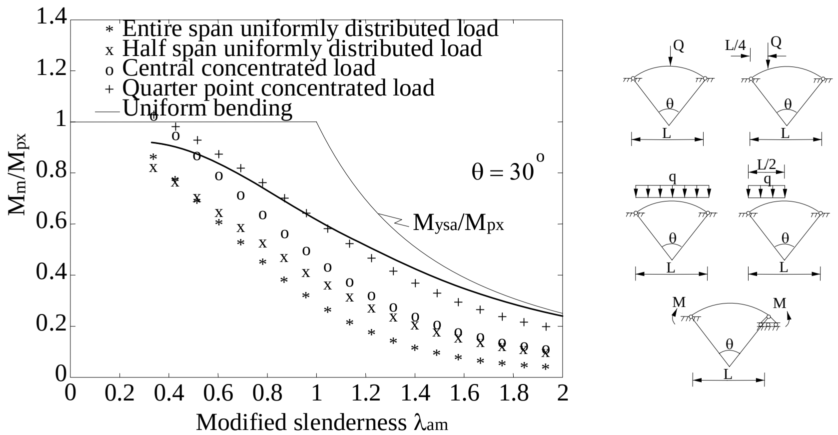


Fig. 5 Effects of loading

The maximum moments M_m of the arches are generally lower than the plastic moment M_{px} of the cross section ($M_m / M_{px} < 1$), except for the arches with small modified slenderness $\lambda_{am} = 0.33$ subjected to concentrated loads, for which the maximum moments M_m are slightly higher than the full plastic moment M_{px} of the cross section ($M_m / M_{px} > 1$), because they fail mainly by yielding and because the pin-ended arches are indeterminate so that their load carrying capacities can continue to increase after the first plastic hinge forms. It can also be seen that the maximum moments M_m of arches with large modified slendernesses λ_{am} are much lower than the full plastic moment M_{px} because they fail mainly by flexural-torsional buckling.

Load-Deformation Behaviour

Typical load -deformation behaviour is demonstrated by the variations of the central twist rotation ϕ_c with the dimensionless maximum bending moment M_m / M_{abx} in Fig. 6 for arches subjected to a load uniformly distributed along the horozontal projection of the entire arch , where M_{abx} are the nominal moment capacities of simply supported arches in uniform bending given by (2) and M_m are the maximum moments in the arches. All the arches fail by a combination of yielding and flexural-torsional buckling, but short arches (with low modified slendernesses λ_{am}) fail mainly by yielding while slender arches (with high modified slendernesses λ_{am}) fail mainly by flexural-torsional buckling.

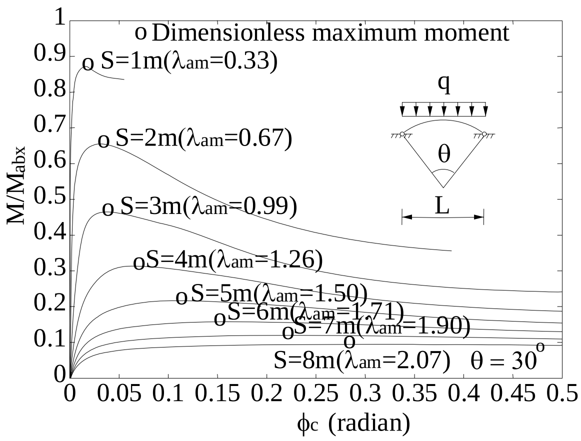


Fig. 6 Load-deformation behaviour

Design

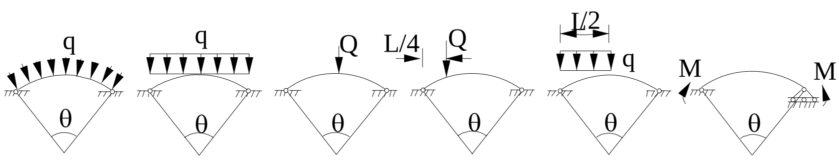
The interaction between bending and compression for arches is related to many factors. This makes it difficult to develop a single general purpose approximation of good accuracy, which is simple enough to be used in design. It has already been found (Pi and Trahair 1998) that the design rules in the Australian Standard 4100 (*AS4100* 1990) for steel beam-columns must be modified before they can be used for the design of steel arches in uniform bending or compression.

In this paper, the following equation is proposed for design against inelastic flexural-torsional buckling for steel I-section arches under combined bending and compression actions

$$\frac{N}{\alpha_{an} N_{ca}} + \frac{M}{\alpha_{am} M_{abx}} \leq 1 \tag{4}$$

in which N_{ca} = the nominal out-of-plane axial compression capacity of an arch in uniform compression, M_{abx} = the nominal moment capacity of an arch in uniform bending given by (1) and (2), respectively, the nominal maximum axial compression N and nominal maximum moment M are calculated by a first-order elastic analysis of the arch, and α_{an} and α_{am} are axial compression and moment modification factors which account for the variations of axial compression and moment along an arch under different loadings. Values of α_{an} and α_{am} are given in Fig. 7.

Inelastic lateral buckling strength and design of steel arches by Yong-Lin Pi and NS Trahair
Address: Department of Civil Engineering, University of Sydney, NSW, 2006, Australia
Phone: 93515217, Fax: 93513343, E-mail: linp@civil.su.oz.au



Arch						
α_{an}	1.0	1.1	1.4	2.7	3.7	—
α_{am}	—	1.1	1.2	1.2	1.1	1.0

Fig. 7 Factors and for arches

The predictions of the proposed interaction equation (3) are compared with the finite element results for arches with subtended angle $\theta = 5^\circ$ to 150° and modified slenderness $\lambda_{am} = 0.20$ to 0.55 in Fig. 8 for arches subjected to concentrated loads, and in Fig. 9 for arches subjected to uniform distributed loads. The proposed interaction equation (3) generally provides satisfactory predictions. The predictions for arches with a moderate subtended angle θ and a moderate modified slenderness λ_{am} are conservative in most cases. The predictions for the arches with a small subtended angle θ and a small slenderness λ_{am} under a load uniformly distributed along a half arch are slightly unconservative.

In some cases, the in-plane strength of a steel arch may be less than its out-plane strength, and so the in-plane strength of the arch should also be checked. The in-plane inelastic buckling behaviour and strength of steel arches have been studied by Pi and Trahair (1996b) and the plastic design of compact steel arches by Trahair et al. (1997).

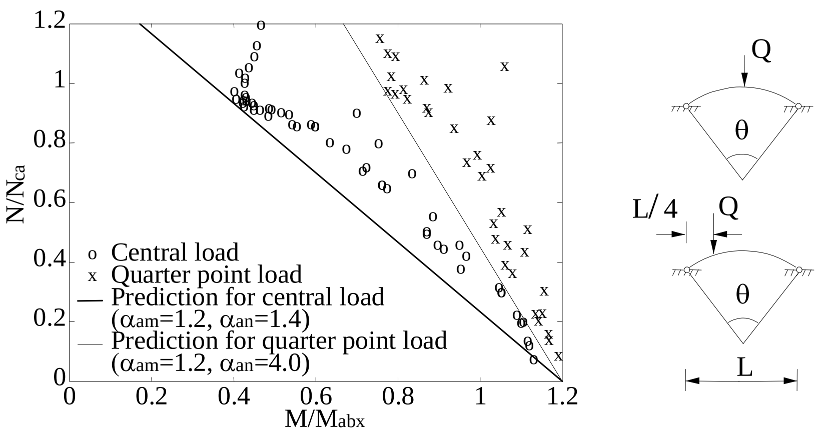


Fig. 8 Strengths of arches with concentrated loads

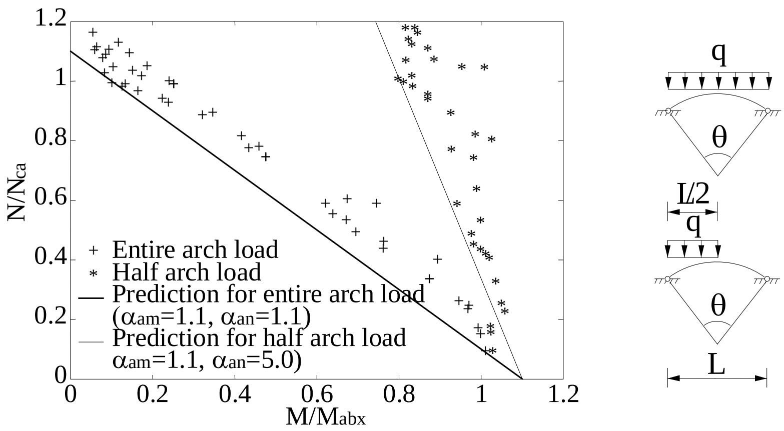


Fig. 9 Strengths of arches with uniform distributed loads

CONCLUSIONS

This paper uses a nonlinear inelastic finite element model for the advanced analysis of arches to investigate the inelastic flexural-torsional buckling strengths and design of steel arches under combined axial compression and bending. The effects of various factors on the inelastic flexural-torsional buckling strengths are also investigated.

It is found that the subtended angle affects the inelastic flexural-torsional buckling strengths significantly. The strength decreases as the angle increases.

The effects of loading on the flexural-torsional buckling strengths are important. The dimensionless maximum moments of the arches subjected to a central concentrated load are generally lower than those of the arches subjected to a quarter point concentrated load. The dimensionless maximum moments of the arches subjected to a load uniformly distributed along the horizontal projection of the entire arch are generally lower than those of the arches subjected to a load uniformly distributed along the horizontal projection of a half arch except for very short arches.

The design rules for steel beam-columns cannot be used directly for the design of steel arches, because they do not include the effects of the in-plane curvature and the subtended angle, but can be modified to include these effects. The proposed design equation (4) generally provides conservative predictions for the flexural-torsional buckling strengths of steel arches.

REFERENCES

AS4100-1990, Steel structures (1990). Standards Australia, Sydney, Australia.

Guide to stability design criteria for metal structures (1988). 4th edition, ed. by T. V. Galambos, John Wiley & Sons, New York, NY.

Komatsu, S. and Sakimoto, T. (1977). "Ultimate load carrying capacity of steel arches." *J. Struct. Div.*, ASCE, 103(ST12), 2323-2336.

Papangelis, J.P., and Trahair, N.S. (1993). “ Design of steel arches against in-plane and -out-of-plane buckling.” *Proceedings*, 12th Australasian Conference on the Mechanics of Structures and Materials, Wollongong University, Wollongong, Australia, 665-672.

Pi, Y.-L. and Trahair, N.S. (1994). "Nonlinear inelastic analysis of steel beam-columns. I. Theory." *J. Struct. Engrg*, ASCE, 120(7), 2041-2061.

Pi, Y.-L. and Trahair, N.S. (1995). "Nonlinear inelastic analysis of arches." *Structure stability and design*, Ed. by S. Kitipornchai et al., A.A.Balkema, Rotterdam, Netherlands, 113-118.

Pi, Y.-L. and Trahair, N.S. (1996a). "Three-dimensional nonlinear analysis of elastic arches." *Engineering Structures*, 18(1), 49-63.

Pi , Y.-L. and Trahair, N.S. (1996b). “In-plane inelastic buckling and strengths of steel arches.” *J. Struct. Engrg*, ASCE, 122(7), 734-747.

Pi, Y.-L. and Trahair, N.S. (1998). "Out-of-plane inelastic buckling and strength of steel arches." *J. Struct. Engrg*, ASCE, 124(2), 174-183.

Sakimoto, T. and Komatru, S. (1983). “Ultimate strength formula for steel arches.’ *J. Struct. Engrg*, ASCE, 109(3), 613-627.

Structural stability design. Steel and composite structures (1997). Ed. by Y. Fukumoto, Elsevier Science, Oxford, England.

Trahair, N.S, Pi, Y.-L., Clarke, M.J., and Papangelis, J.P. (1997). “Plastic design of steel arches.” *Advances in Structural Engineering*, 1(1), 1-9.

Yang, Y.-B. and Kuo, S.-R. (1987). "Effect of curvature on stability of curved beams." *J. Struct. Engrg*, ASCE, 113(6), 821-841.