Hybrid Cable-Arch Systems
for Long Span, Lightweight Roof Structures

by

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ABSTRACT:

This paper discusses the behavior of hybrid roof structures made by prestressing a system of intersecting arches using an under-slung linked secondary cable system. The effect of the cable system is to diffuse the loads applied to the arches thus promoting a structure which can be thought to be fully utilized in terms of strength. A comparison is made between a prestressed and a nonprestressed system to illustrate this behavior. We will discuss how the hybrid system of arches and cables is essentially a “prestressed shell” when taken in its composite form. We will also discuss and quantify the reduction of localized moments in the arches as they are effected by the cable prestress. Other benefits (reduction of support reactions, force flow, etc…) associated with optimizing the prestress are also addressed. The ongoing

roof replacement for the UNI-DOME, an athletic facility with a 450' span in Cedar Falls, Iowa, USA, is used to illustrate the points discussed.

INTRODUCTION:

The hybrid system discussed here was developed to replace an existing air supported roof system at the University of Northern Iowa (UNI-Dome). This paper will touch upon some of the existing conditions that influenced the structural design. However the focus will remain on the overall behavior of the system.

The hybrid system designed for this application utilizes the existing cables of the UNI Dome, a 450ft diameter air supported roof, in their in-place location, as a secondary cable system linked by vertical members to a new crossed arch system. Stainless steel, standing seam roof panels supported by structural metal deck and bar joists form the skin of the peripheral area of the roof, covering 75% of the roof surface. The center 45,000 SF polygon is enclosed with an arch supported (PTFE) fabric tensile roof. See Figure 1 for a computer rendering of the completed UNI-Dome project.

The roof structural design is broken up into two main sections, the main roof section and the fabric skylight section. The main roof supporting system is composed of a crossed (box section) arch roof system and a secondary cable system. The crossed arch system is made up of four (4) main arches and sixteen (16) secondary arches, which when assembled, provide the skeleton of the structure. The secondary cable system that is located below and along the plan center–line of the arches, in addition to resisting uplift, cause the crossed arch system to act as a prestressed shell. Figure 2 shows the hybrid arch-cable structure system of the UNI Dome.

HYBRID STRUCTURAL BEHAVIOR:

The structure is complex to the extent that the secondary cable system reacts in a nonlinear manner and can not be analyzed using linear methods, just as a cable net's behavior of exhibiting large deflections and small strains requires a nonlinear analysis. Figure 3 and 4 show the geometrically nonlinear behavior that is typical of hybrid systems. Figure 3 shows the classical stress stiffening that is common in cable nets. That is, in this example the total structural stiffness is created by prestressing the structure. The stiffness (K) clearly increases linearly as the prestress (F) increases. Applying this classical stress stiffening behavior to an elastic structure we get Figure 4, which is more characteristic of the system under discussion. The structural stiffness that is now created is the summation of the geometric stiffness (K_1), the elastic stiffness of the arches (K_2) and the addition of stiffness due to a prestress that compresses the arches (K_S).

The addition of prestress, which increases the stiffness of the arch system, benefits the structure in several ways. By loading the structure in compression the arch action behavior of the system is increased thereby increasing stability, decreasing deflections and reducing the effects of localized bending.

There are two components to the system response. Under symmetric loading, such as dead load, there is a strong participation by the cable system. In this case the geometry conspires to give very little bending moment but the arch is forced to carrying most of the load as thrust since it is relatively stiff when compared to the cable system. For unsymmetrical loads such as dead load plus live load over half of the structure the cable system does not relieve the arches of moment. It behaves very similar to an arch that is experiencing an unbalanced loading. However, as the cable net changes geometry to maintain equilibrium, under an unbalanced case the distribution of horizontal forces remains more uniform than if the system was not stressed. As a result, the uniform horizontal thrust relieves the amount of bending the perimeter structure experiences.

The behavior of the three-dimensional dome structure is similar to that of a shell or simply taken as a two dimensional arch. For discussion purposes the interaction between the crossed arches will be ignored to illustrate the behavior of the hybrid system. Ignoring the effects of prestress on the cable /arch system shown in Figure 5, the arch illustrated, is funicular for only one load condition (a dead load condition); all other loading conditions introduce bending in the arch. Bending occurs when an unbalanced load is imposed on the arch and the arch is unable to change shape to remain funicular. The top chord is compressed and the bottom chord is put into tension. When the arch is stressed by the presence of a linked cable system, both the top and bottom chords are put into compression. The simple compression of all chord members is the essence of arch action. By creating prestressed arch action the capacity of the system to resist bending is increased. Given an unbalanced loading condition as illustrated in Table 1, for a stressed system and an unstressed system, Table 2 gives the resulting forces in the center chords of the arch in Figure 5. The unstressed system exhibits tension in the bottom chords while the stressed system remains in compression. It should be noted that the axial contribution from the prestress should be optimized so as not to produce diminishing returns when designing arch members.

As a result of building a compression force, that increases stability, into an arch system, deflections are naturally effected. Table 3 illustrates the effect of an increasing incremented load, on three interior arch nodes, for a stressed and unstressed system. Initially, the stressed system exhibits larger deflections due to elastic shorting. However as an increased load is added to both systems, a decrease in relative deflections of the stressed case is noticed when compared to the unstressed case. The stressed system demonstrates an increase in deflection (over the range of loading shown in Table 3) of approximately 31%, while the unstressed system yields a 46% increase.

The shape of the arch is directly influenced by the shape of the existing cable. An optimization of the arch / cable separation was conducted to reduce the horizontal thrust on the perimeter of the structure. In other words, the greater the separation between the cable and the arch, in elevation, the less horizontal thrust is exerted on the perimeter of the structure by the arches. Table 4 illustrates the effect the cable net has on the existing perimeter structure. It is clear that a cable net can drastically minimize the horizontal thrust produced by an arch. In addition, as previously noted, the arch geometry was designed to be funicular under a dead load plus prestress case, likewise, the prestress force in the cable net and the geometry of the arches was optimized to obtain a uniform loading on the perimeter of the structure.

The spatial nature of the dome offers an interesting advantage with regard to hybrid behavior. While a load applied to a single arch has a fairly direct path to the supports, in a dome where arches interact the load path is more complex and a load can diffuse along the surface of the arch. This behavior combined with the influence of prestess explains the reduction of localized bending moments outlined in the following table of maximum dead plus live load bending moment.

	Fraction of Maximum Bending Moment	
	Prestressed case	Unprestressed case
One Way Arch System	90%	100%
Two Way Arch System	80%	100%

DESIGN OF THE UNI-DOME:

The above discussion has outlined the general behavior of a hybrid system as developed for the UNI-Dome. This section will go into some of the more specific design considerations that influenced the application.

The original design of the UNI-Dome was completed in 1975. It was the first of eight stadium size air-supported fabric roof system based on the concepts developed by the late David Geiger. This technology enabled roof construction costs to be drastically reduced and the time to build a stadium was, in some cases, cut in half. For many years air-supported roof structures replaced conventional, rigid roof structures entirely for stadium size roof covers.

Operating stadiums with air-supported roofs was not as simple as expected. Stability of the roof depended on the interior pressure to be larger than the exterior load. The design of walls, doors and windows limited the design pressure to not much more than 5psf. Design snow loads in locations such as northern Iowa is as large as 40 psf. Hot air directed to the roof surface would not always be sufficient to melt the snow fast enough to retain this necessary balance. Consequently, deflation occurred, in a few cases causing deflation and law suits. Manual snow removal procedures adopted by users of air-supported roofs became the way of taking care of the problem.

When the UNI-dome roof deflated in December of 1994, the University asked Light structures Design Consultants of White Plains, NY (a subsidiary of DeNardis Associates) to study alternative designs for the replacement of the roof. The scheme presented here was adopted and developed for it's functionality and aesthetic value.

The hybrid design is based on utilizing the ingenuity of the existing roof geometry and on making use of as much of the existing structural components as possible. The roof replacement design was controlled by the existing structural condition , configuration as well as the desires of the owner. The hybrid system uses the existing cable net, columns, and a reinforced concrete circumferential girder. The existing concrete compression ring is converted into a tension ring by prestressing the structure's periphery with a post tensioning system of tendons. The existing cable net, connected and stressed against the arches, gives the arches stability and allows them to be slender and relatively light weight. All structural components are shop fabricated and shipped to the site in segments which are assembled by bolted connections. The center skylight was designed to have nearly the same translucency to the roof as the system it is replacing. To aid in an energy savings, the opaque roof area is was design with insulated roof assembly.

CONCLUDING REMARKS:

The hybrid cable / arch design described in this paper is unique in many ways. Most particularly, it is a symbiosis of conventional roof technologies and the more contemporary light-weight, long span technologies of fabric roofs. It brings together the best of these two schools. The "skylight" section, for example, is both architecturally appealing and cost effective.

There is considerable potential in hybrid systems which remains to be explored. For example, rather than given as it is in the case of the UNI Dome retrofit, the cable geometry can be designed for a specific application, allowing both the magnitude and the spatial distribution of the prestress to be varied. It would then be possible to use an initial state of flexural prestress on the arches rather than simple axial load as had been done here. It would also be possible to investigate the effect of cables going slack in some conditions of extreme loading giving a structure whose behavior would change in response to the type and magnitude of loading.

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Figure 1 – Computer Rendering of Completed Roof Superimposed on Photo.

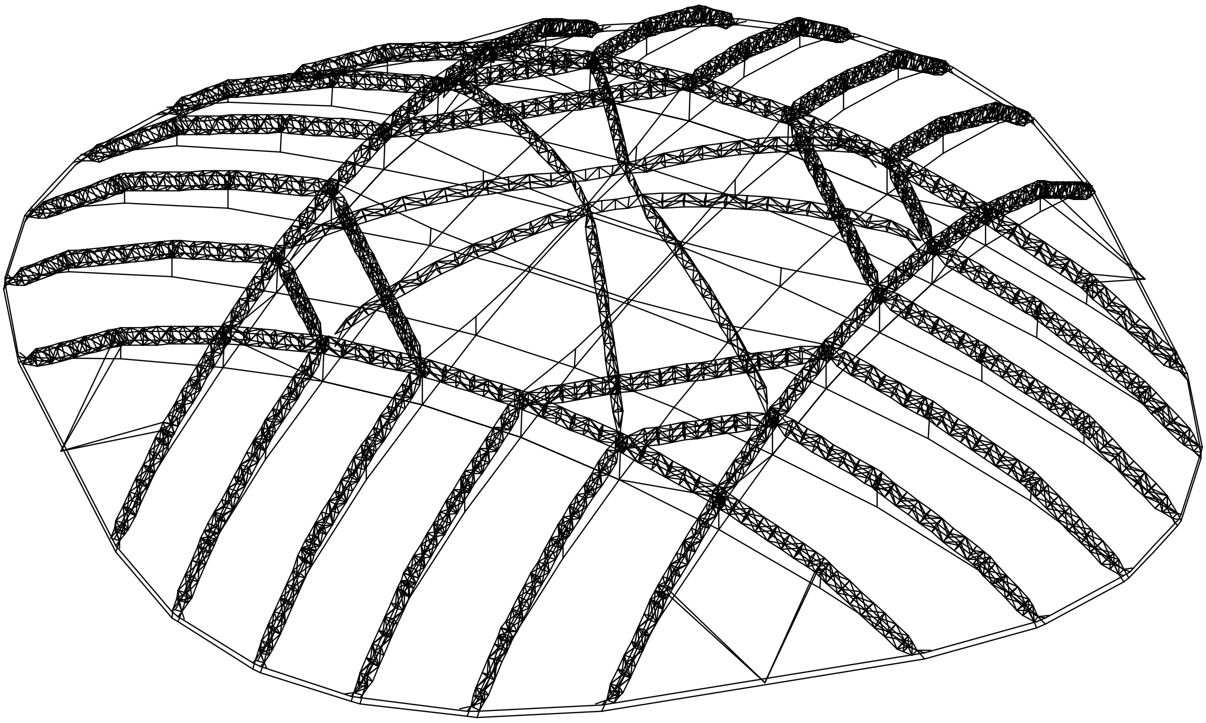
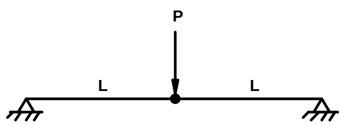
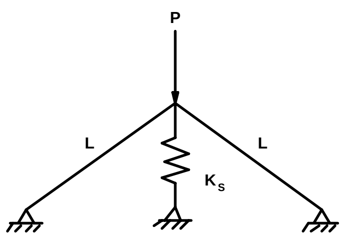


Figure 2 – Computer Model of Structural Components.



Stiffness: K = 2F/L
F = Prestress

Figure 3 – Stress Stiffening



Stiffness: $K = K_1 + K_2 + K_s$ A = Bar Area
E = Young's Modulus

Geometric Stiffness: $K_1 = 2F/L\ (1 – \sin^2 Ø)$
Elastic Stiffness: $K_2 = 2AE/L\ (\sin^2 Ø)$

Figure 4 – Hybrid Structural Behavior.

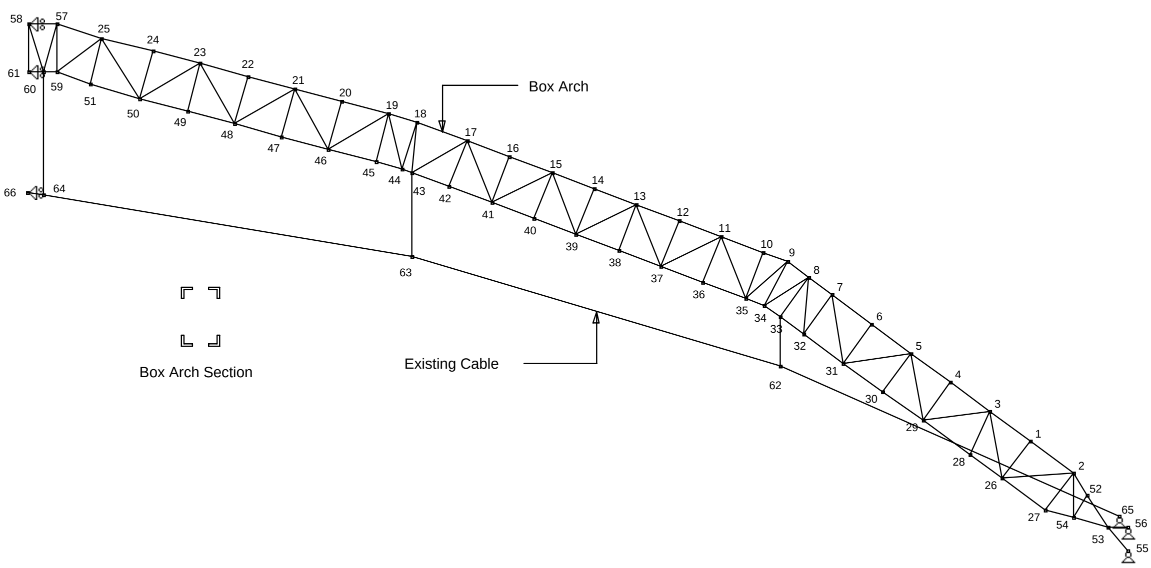


Figure 5 – UNI-Dome Hybrid Arch / Cable System

Figure 6 – UNI-Dome, Under Construction 6/18/98

Table 1	**Applied Loads(lb)**
Node	Pz
9	-5500
10	-5500
11	-5500
12	-5500
13	-5500
14	-5500
15	-5500
16	-5500
17	-5500
18	-5500
Total	-55000

Table 2		**Member Force(lb)**	
		Stressed System	Unstressed System
Start Node	End Node	Cable Force =190,000 (lb)	Cable Force =0 (lb)
12	13	-149709	-113048
13	14	-170970	-111046
14	15	-168970	-117350
15	16	-170957	-102660
37	38	-25633	43000
38	39	-25633	43000
39	40	-12021	41821
40	41	-12021	41821

Table 3		**Deflections (ft)**	
Total Load	node	Stressed System	Unstressed System
-45000	14	-0.143	-0.102
	15	-0.147	-0.096
	16	-0.152	-0.089
-55000	14	-0.166	-0.125
	15	-0.169	-0.118
	16	-0.172	-0.109
-65000	14	-0.188	-0.149
	15	-0.19	-0.14
	16	-0.192	-0.129

Table 4	**Support Reactions at Ring Beam**			
	Stressed System		Unstressed System	
Node	Rx (lb)	Ry (lb)	Rx (lb)	Ry (lb)
55	110369	-126052	48148	-55010
56	61308	72	18986	10
65	-161110	70980	0	0
Total	10567	-55000	67134	-55000