The Design Critical Load and Safety Coefficient of Reticulated Shells

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Abstract

In this paper the factors that influence the determination of theoretical critical load are briefly discussed. Having made a comparison of some formulas for evaluating the design critical load of RC shells, the authors present the expression of design load for design of reticulated shells on linear or nonlinear theory and provide the safety coefficient as well. The available range of the formulas on classic linear theory are particularly discussed. The practice engineering example shows that the formula and the safety coefficient provided in the paper are reasonable and advantage in practice.

1 Introduction

In recent several decades many research on the instability theory and nonlinear instability analysis method of reticulated shells have been carrying on in world, although a lot of single layer dome wear successfully designed. It is no doubt that the nonlinear incremental FEM is a most widely used analysis method for tracing the equilibrium path of a structure under loading and detecting critical point, consequently, to determine critical loading. However, a more practical formula is demanded to provide the estimation of critical loading and to give an appropriate safety factor as well for designing the reticulated shells. Having studied and compared with different analysis methods and expressions of critical loading in practice, the authors present a formula to give an approximate design critical loading of the reticulated shells.

2. Factors for Influencing the Critical Loading of a Structure

Some studies wear carried on the instability mechanism of the reticulated shells in order to find out the factors that influence the critical loading. So far it focuses people's attention on theoretical and experimental research of instability of structures. There is a great extent difference in critical loading derived from theoretical analysis and testing, respectively. It took several decades for people to look into the cause which make the difference. It may find out that the most experimental results of buckling load of the structures are extremely dispersive but not all. Two model testing of spherical dome wear performed by Prof. R. E. McConnel in Cambridge. The models were meticulously made with bars. There were 19 nodes and 42 members in the dome. The reduction of buckling load due to the geometry deviation is 4 percent of that resulted from theoretical analysis, while only a concentrate load applies to the top of the dome. It appears quite different as 7 loads apply to the dome simultaneously. The imperfection makes the reduction of buckling load in 40 percent. In this model testing only geometry error were in consideration. Although very few papers regarding experiment with meticulously made models and theoretical analysis were presented so far, the conclusion that only imperfection makes the reduction of buckling load could not be derived. If all the presented results of both experimental and theoretical research are put together and compared, it may be found that the difference of critical load in a great extent between experimental results and theoretical analysis is caused by many factors. The main influence factors include

(1) the theoretical mathematics and mechanics analysis model,
(2) the algorithm for tracing the equilibrium path in the duration of loading,
(3) the initial geometry shape of the structure,
(4) the loading pattern applying to the structure,
(5) the imperfection type existed in the structure,
(6) the boundary condition of the structure.

The configuration and initial geometry shape of the structure, the boundary condition and extent of restriction of the supports, the loading and imperfection pattern could influence and change the structure mechanical feature, buckling pattern and critical loading indeed. The theoretical mathematics and mechanics analysis model and the algorithm for tracing the equilibrium path in the duration of loading could not exactly express and mention the structural character and behaviors. There is a difference between the boundary condition and extent of restriction of the supports in practice and that in theory. People never looks down on the error from expression of the boundary condition and restriction of the supports base upon the ideal assumption.. The influence of imperfection to both buckling pattern and critical loading has been accepted as main factor since 60's. There are several imperfection patterns, in which the initial stresses in the structure behave a lot to reduce the buckling loading. The reticulated shell has no ability for redistributing the unbalanced initial stresses and relaxing by changing the configuration or geometry shape itself as a flexible structure. The unbalanced initial stresses caused by different imperfection produce the strong power to alter structural geometry. The most widely studied imperfection so far involves initial deviation of structural geometry only. Some assumptions and methods are presented for tracing the equilibrium path to pursue post buckling point and relevant loading. In fact, the so called initial deviation of structural geometry is the deformation of the structure behaved in process of redistributing the unbalanced initial stresses. It is the initial stress but geometry make the buckling pattern changed and buckling loading reduced. Therefore, all of the imperfection pattern have to be taken into account

In practical engineering design, the engineers are interesting in the actual bearing capacity of a structure while buckling and the permitted design critical loading with an appropriate safety. The permitted design critical loading would be derived from a certain reduction of the theoretical critical loading. It is obviously significant to modeling the imperfection pattern, seeking for a nonlinear mathematics and mechanics model and an algorithm for tracing the equilibrium path. A nonlinear analytical model was provided in ref. 【1】, in which some of imperfections, axial and flexible initial stresses are taken into account in terms of expressions. It is possible to detect the critical point and consequently evaluate the theoretical buckling load. The design critical load is expressed in terms of theoretical buckling load multiplied by a series of reductions.

3 The Design Critical Load and Safety Coefficient for RC Shells

It has no sufficient evidence from experiment of reticulated shells to verify the rationalization of design critical load, because only few testing have been performed. Most experimental and theoretical researches had been done involve the domain of RC shells. Fig 1 shows the buckling loading Of RC shells which were from the experiments performed several years ago in the world.

All these conclusion for RC shells are very much valuable in case very few experimental research of reticulated shells are available. It reveals a way to formulating the design critical loading and safety coefficient of reticulated shells from that of RC shells. Therefore, it is necessary to discuss the formula of critical loading of RC shells first.

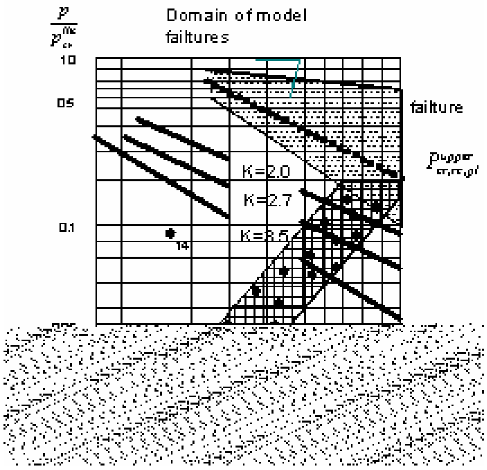

Fig 1.experimental results of buckling load of RC spherical shells

(1) Formula recommended by IASS

IASS fifth Working Group provided this formula. It is from the achievement of Prof. Kollar and Dulacska. This formula is available for practice and conservative as well.

The first step in this method is to evaluate the bifurcation load of a spherical shell under radius loading by using classic theory.

$$P_{cr}^{bif} = \frac{2E}{\sqrt{3(1-\mu)^2}}\left(\frac{h}{R}\right)^2 = 1.2E\left(\frac{h}{R}\right)^2 \qquad (1)$$

The second step is to evaluate the design critical load in multiplying the bifurcation load by a series of reductions.

$$\gamma_1 P_{cr}^{des} = a_1 a_2 a_3 a_4 P_{cr}^{bif} \quad (2)$$

In which

γ_1 Implies to safety coefficient

a_1, a_2, a_3, a_4 Are the reductions for imperfections. a_1 is a sensitivity of imperfection. There is a relationship between a_1 and the radio $\frac{w_0}{h}$. Here, h is the thickness of RC shell and w_0 is the measured imperfection. One suggestion is to assume w_0 being the summation of the deflection w_1 and the initial imperfection w_2. It approximately equals to the thickness of shell h_0. The further suggestion from Prof. Medwadowski is to ignore the deflection w_1 and the initial imperfection w_2 is limited within a range.

$$\frac{h}{3} < w_0 < \frac{h}{2}$$

a_2 is a creep factor. The creep factor a_2 is a ratio of creep elastic modus Ecr and RC elastic modus Ec.

$$a_2 = \frac{E_{cr}}{E_c} \quad (3)$$

a_3 is a crack factor.

a_4 is a RC non elastic factor.

γ_1 is a safety factor. It may be set as 3.5 for a sensitive structure and 1.75 for other structure.

(2) Formula recommended by ACI

The design critical loading formula recommended by ACI is as follow.

$$\gamma_2 P_{cr}^{des} = K_1 E\left(\frac{h}{R}\right)^2 = \frac{K_1}{1.2} P_{cr}^{bif} \quad (4)$$

in which

γ_2 is a safety factor. In general, $\gamma_2 > 4.0$

K_1 is a reduction factor. As the ratio of rise and span is in the range of 1/6 o 1/10, $K_1 = 0.25$. If $K_1 = 0.25$ and $\gamma_2 = 5.0$, hence

$$P_{cr}^{des} = 0.05E\left(\frac{h}{R}\right)^2 \quad (5)$$

(3) Formula recommended by BJG 16-65

The China design code BJG 16-65 provides a formula for evaluation of design critical loading of the revolution shells. The recommended formula is

$$\gamma_3 P_{cr}^{des} = 1.2K_2 E\left(\frac{h}{R}\right)^2 \quad (6)$$

If $K_2 = 0.25$ and $\gamma_3 = 5.0$, hence

$$P_{cr}^{des} = 0.06E\left(\frac{h}{R}\right)^2 \quad (7)$$

It would be clearer in comparing these formulas with a numerical result. Table 1 shows the design critical loads of a RC spherical dome which are derived from these formulas. The span of the dome is 37.5m, the thickness h = 8.0cm,the elasticity modus $E = 3.0 \times 10^7 kN$

Design Critical Load and safety coefficient Table 1.

ANALYTICAL MODEL	IASS FORMULA	ACI FORMULA	BJG16-65 FORMULA
	$\gamma_1 P_{cr}^{des} = a_1 a_2 a_3 a_4 P_{cr}^{bif}$	$\gamma_2 P_{cr}^{des} = 0.25E\left(\frac{h}{R}\right)^2$	$\gamma_3 P_{cr}^{des} = 025 P_{cr}^{bif}$
Safety Coefficient	3.5	5.0	5.0
Design Critical Load $\gamma P_{cr}^{des} (kN/m^2)$	6.044	34.133	40.96
Permit Design Load $P_{cr}^{des} (kN/m^2$	1.727	6.829	8.192
error	1.0	3.95	4.143

It is obvious from the Table 1 that the design critical loads derived from the formulas are quite different. All the formulas are base on the bifurcation load provided with classic linear theory.

4. **The Design Critical Load of the Reticulated Shells on Non–linear Theory**

The design critical load could be expressed with theoretical buckling load multiplying a series of reduction coefficient, which the buckling load is obtained by using nonlinear stability analysis theory. The design critical load is given by

$$SP_{cr}^{des} = \beta_1 \beta_2 \beta_3 \beta_4 P_{cr}^{nl} \quad (8)$$

In which

S Implies to safety coefficient

β_1 is a reduction parameter and implies to the sensitivity of initial imperfection.

β_2 is a reduction parameter and implies to the action of creep, in general β_2=1.0

β_3 is a reduction parameter for dynamic action.

β_4 is a reduction parameter and implies to the response of loading.

P_{cr}^{nl} is the critical load of an ideal system, which is derived with nonlinear instability analysis theory. As mentioned above, the configuration, the boundary condition and the loading and imperfection pattern are taken into account in the nonlinear incremental FEM. It is more important to estimate the sensitivity of imperfect.

The initial imperfection reduction parameter could be expressed as

$$\beta_1 = \xi_1 \xi_2 \xi_3 \xi_4 \tag{9}$$

Here

ξ_1 is a factor of initial deviation of geometry. The initial deviation of both geometry and loading is considered with this factor. ξ_1 could be set as a range of 0.6 to 0.65.【2】,【3】, or directly obtained by analysis.

ξ_2 is a factor of initial stresses in the structure. The initial stresses would be produced in assembly process due to error in length of members and assembly procedure, methods etc. The initial stresses could not avoid and ignore. ξ_2 could be set as a range of 0.9 to 0.8..

ξ_3 implies to a factor regarding imperfection in materials. Normal, ξ_3=0.95.

ξ_4 implies to a factor regard softening velocity of structural stiffness in the duration of loading. It reveals the nonlinear feature of imperfection. ξ_4 could be defined with stability factor K_s in the i Th and $i+1$ Th iteration , 【2】【3】, and expressed as

$$\xi_4 = \left(\frac{K_S^{I+1}}{k_S^I} \right)_{aver}$$

ξ_4 could be set as a range of 0.9 to 0.95, in case it may not evaluate K_s directly.

All the imperfection influence should be taken into account simultaneously. However, the response of imperfections is assumed as independence. The suggested values of factors could not be beyond a permitted range.

5. Numerical solution

A reticulated shell with a span of 47.2m and a radius of 26.4m in Sanxi was carried on nonlinear instability analysis. Equation (8) was adopted to check the critical load. The factors were set as

$$\xi_1 = 0.62,\ \xi_2 = 0.9,\ \xi_3 = 0.95,\ \xi_4 = 0.9$$

$$\beta_1 = 0.47709,\ \beta_2 = 1.0,\ \beta_3 = 0.95,\ \beta_4 = 0.9$$

The safety coefficient $S = \frac{S_0}{1 - a\gamma} = \frac{1.25 \times 1.234}{-2.0 \times 0.2738} = 3.4096$

The design critical load is

$$SP_{cr}^{des} = 0.47709 \times 1.0 \times 1.0 \times 0.9 P_{cr}^{nl} = 0.429381 P_{cr}^{nl}$$

The permitted design load is $P_{cr}^{des} = \frac{0.428381}{3.4096} P_{cr}^{nl}$,

The expecting critical load is $7.9407 kN/m^2$. The theoretical buckling load is $8.2 kN/m^2$.

6. Remarks

Most of factors influencing the buckling load of a structure are taken into account in the formula provided in this paper. Numerical solution shows that it is able to use in practical design for estimating the permitted design load.

Reference

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