The Analysis and Construction of A Three-eighths Single Layer Reticulated Dome

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ABSTRACT

This paper presents a theoretical analysis and construction of a three-eighths single layer reticulated dome which is built with cold-worked thin-walled section steel. The calculation results by frame FEM analysis and the design according to the Chinese Code is given. The shell stability is analyzed by pseudo-shell method based on the continuous mechanics model. Finally, a brief introduction to the construction is given.

Reticulated shell structure is a both ancient and young style of structure. It is also a favorite structure because its beautiful shape and good structural properties. The most common reticulated shell is spherical shell (dome). Thin-walled section steel is an economical steel shape. It can increase the efficiency of the definite material by optimizing the shape of the section. It has great potentiality to combine the reticulated shell with the thin-walled section steel.

1. INTRODUCTION

This reticulated shell has been built in Jiangsu Province in East China. It is a motion-picture screen for projection in an cinema. The spherical shell has the diameter of 23 meters, the horizontal azimuth of 180 degrees, elevation angle of 138.4 degrees, almost three-eighths of 360 degrees, as shown in Fig.1 and Fig.2. The whole dome is supported by ten steel columns. The height of the columns are from 3m to 7.6m. The peak of the shell is 23.1m high. There is a cylindrical shell in front of the spherical shell to act as the lateral support of the spherical shell. The audience hall is just below the cylindrical shell.

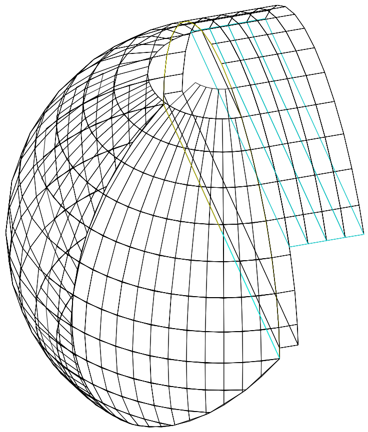


Fig.1 Vertical View

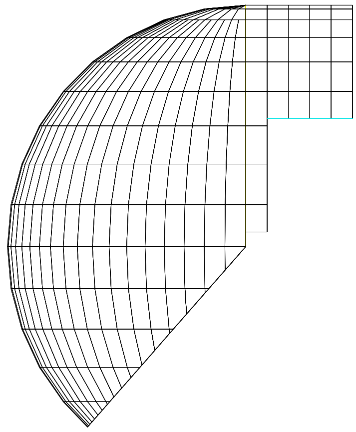


Fig.2 Sectional Elevation

The spherical shell is of Schwedler, double diagonal members. There are all together 14 rings from the bottom of the shell to the top of the shell. All diagonal members in the same ring is of the same length, so be all latitudinal members and longitudinal members. A latitudinal member is faced with a spherical angle of 5 degrees.

2. DESIGN OF THE SPHERICAL SHELL

2.1 FEM Theory

The frame FEM is used to analyze the spherical shell. The element in the calculation model is a three-dimensional uniform beam. The displacement vector of the node in the local coordinate system is

$$u_e = \left[u_i \; v_i \; w_i \; \theta_{xi} \; \theta_{yi} \; \theta_{zi} \; u_j \; v_j \; w_j \; \theta_{xj} \; \theta_{yj} \; \theta_{zj}\right]^T$$

the relative node force vector is

$$p_e = \left[p_{xi} \; p_{yi} \; p_{zi} \; m_{xi} \; m_{yi} \; m_{zi} \; p_{xj} \; p_{yj} \; p_{zj} \; m_{xj} \; m_{yj} \; m_{zj}\right]^T$$

the FEM equation for the space beam in the local coordinate system is

$$k_e u_e = p_e \qquad (1)$$

where, k_e-- the elastic rigidity matrix for the beam in the local coordinate system.
In the global coordinate system, the displacement vector is

$$U_e = \left[U_i \; V_i \; W_i \; \theta_{xi} \; \theta_{yi} \; \theta_{zi} \; U_j \; V_j \; W_j \; \theta_{xj} \; \theta_{yj} \; \theta_{zj}\right]^T$$

the relative node force vector is

$$P_e = \left[P_{xi} \ P_{yi} \ P_{zi} \ M_{xi} \ M_{yi} \ M_{zi} \ P_{xj} \ P_{yj} \ P_{zj} \ M_{xj} \ M_{yj} \ M_{zj} \right]^T$$

by transformation

$$u_e = RU_e \quad (2)$$

$$p_e = RP_e \quad (3)$$

the basic equation in the global coordinate system can be obtained by substitution (2) (3) into (1):

$$K_e U_e = P_e$$

where, K_e -- the elastic rigidity matrix for the beam in the global coordinate system,

$$K_e = R^T k_e R$$

R -- the transformation matrix, which express the relation between the principle axis system of the section, the vectors defined in the local coordinate system and the vectors defined in the global coordinate system.

2.2 Boundary Conditions

The columns at the bottom of the spherical shell can be considered as fixed restraints, because it has great rigidity. The cylindrical shell in front of the spherical shell acts as an elastic restraints. The rigidity along the elastic restraints is taken into account by adding the rigidity on the corresponding diagonal element in the global matrix.

2.3 Results

In the Chinese Code "Code for Cold-working thin-walled section steel structure"(GBJ18-87), the member can be designed as follows:

$$\sigma = \frac{N}{A} \pm \frac{M_X}{W_X} \pm \frac{M_Y}{W_Y} \leq f$$

where, N -- axial force;

M_X, M_Y -- the moment about principal axis of the section;

W_X, W_Y -- the section modulus about the principal axis;

A -- area of section;

f -- yields strength, 235N/mm^2.

For the convenience of construction, the members are categorized by the FEM calculation results. The maximum internal forces combination for the longitudinal and latitudinal members is

$$N = \text{-7583 N} \qquad M_X = \text{-6.5 N-m} \qquad M_Y = \text{-14.7 N-m}$$

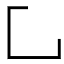
The maximum internal forces combination for the diagonal members is

$$N = \text{-3015 N} \qquad M_X = \text{-7.6 N-m} \qquad M_Y = \text{-3.6 N-m}$$

For first combination, the section steel $C80\times40\times15\times2.0$ is selected, while $L40\times15\times2.0$ is selected for second combination. The characteristics of the sections are shown in table 1. According to the above equation, the maximum stress for longitudinal and latitudinal members is $\sigma = 37$ N/mm^2 $< f$, while that for diagonal members is $\sigma = 30$ N/mm^2 $< f$.

The maximum reaction force is 9.015 KN and the maximum vertical displacement is only 2.1mm. The results also shows that the moment in the supports is greater than that away from the supports, which results from the irregular boundary condition.

Table 1

type	$C80\times40\times15\times2.0$	$L40\times15\times2.0$
shape		
characteristics	$A = 3.47$ cm^2 $m = 2.72$ kg/m $I_X = 34.16$ cm^4 $I_Y = 7.79$ cm^4	$A = 1.95$ cm^2 $m = 1.53$ kg/m $I_X = I_Y = 3.93$ cm^4

2.3 Design of the Connection

All connections used welding. Welding electrode is of E43 which has equivalent strength to the basic material.

The weld bead height is 3mm and the thickness of the connecting plate is 8mm.

3. SHELL STABILITY ANALYSIS

The stability of a single-layer spherical shell is an important problem in the structural analysis of reticulated shell. The design of most single-lager reticulated shell is controlled by the shell stability. But it is very difficult to get the exact solution of the shell-stability of the single-layer reticulated shell. A pseudo-shell method on the basis of the continuous mechanical model is used to analyze the shell stability. The single-layer spherical shell consisting of beams are compared as a continuous thin shell with equivalent rigidity. The analysis of this continuous model by means of the classical theory of solid shell can get the low critical load.

Referring to the new achievement on the stability of spherical shell and considering the characteristics of the actual structure, the following equation is used to calculate the low critical load.

$$P_{cr}^{low} = CP_{cr}^{lin} \qquad C = 0.3 \qquad P_{cr}^{lin} = \frac{E}{R^2}\sqrt{t\delta^3}$$

where, P_{cr}^{lin} -- linear critical load of the spherical shell;

E -- elastic modulus;

t -- membrane thickness of the equivalent continuous spherical shell, $t = \frac{2\overline{A}}{\sqrt{3}l}$;

δ -- average value of the bending thickness, $\delta = (20.785\frac{\bar{I}}{l})^{1/3}$;

$\overline{A}$ -- average area of all members of the reticulated shell;

l -- average length of all members of the reticulated shell;

$\bar{I}$ -- average moment of area of all members of the reticulated shell;

R -- curvature radius of spherical shell.

The low critical load can be obtained by replacing all structural parameters in above equation with actual data.

$$P_{cr}^{low} = 2.4 \text{ KN/m}^2$$

the design load is 0.25 KN/m^2

the safety factor $K = \frac{2.4}{0.25} = 9.6$, which can be thought as reliable.

4. INTRODUCTION TO THE CONSTRUCTION

All members are cold-worked with Q235 thin plate of 2.0 thickness. The steel has been galvanized before cold-working. A full scale rigid foetal membrane is built in the workshop. All members are worked as the required curvature according to this membrane and then transported to the site. Full framing is set up for welding at the site. The input of heat is controlled strictly in the welding to avoid the basic material melting through and lowering the steel strength. The welding is in a reasonable sequence to reduce the welding deformation. Visual examination and ultrasonic inspection to all weld demonstrates that they all meet the code. Some temporary bracing are used to stable the members and ensure the exact positions of all members during erection. The erection begins with the middle support and extends to both sides, and from bottom to top. This can reduce the welding shrinkage stress as great as possible.

The screen plates are fixed on the reticulated shell by riveting after the erection. The site survey shows that the error of curvature radius is only 5mm.

All construction lasted for one and a half month.

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