ANALYSIS OF CRITICAL STATES OF SPACE GRIDS COVERING LARGE SPACE

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Abstract - The paper presents a proposed extension of the numerical handling, developed by us for single-layer or double-layer pin-jointed space grids, to space frames with nodes of rigid connection. The method is suitable for numerical description of local and global loss of stability of bar structures supported along the boundary and covering very large area, or of other large bar structures having usually many redundant bars.

NOMENCLATURE

b	number of bars
$\mathbf{B}_i$	transfer matrix of bar i
j	number of internal nodes
j_1	number of external nodes
j_0	number of all nodes: $j_0 = j + j_1$
h_i, k_i	signs of the starting point and of the end point of bar i (i = 1, 2,..., b)
l_i	length of bar i
$\mathbf{K}$	stiffness matrix of the structure
$\mathbf{F}$	hyperdiagonal matrix composed of matrices $\mathbf{F}_i$
$\mathbf{F}_i$	flexibility matrix of bar i
$\mathbf{G}$	equilibrium matrix of the structure
$\mathbf{q}$	hypervector of loads on the structure
$\mathbf{q}_c$	6-dimensional vector of generalized load applied at node c
$\mathbf{Q}$	flexibility matrix of the structure
$\mathbf{r}_c$	generalized position vector of node c (c = 1, 2,..., j_0)
$r_{c,n}$	nth component of generalized position vector of node c (n = 1, 2,..., 6)
$\mathbf{s}$	hypervector of forces in bars of the structure
$\mathbf{s}_i$	$\equiv \mathbf{s}_{ki}$ 6-dimensional vector of generalized force in bar i
$\mathbf{t}$	hypervector of kinematic loads on the structure
$\mathbf{t}_i$	6-dimensional vector of kinematic load on bar i
$\mathbf{T}_i$	6×6 matrix making transformation between the local (belonging to bar i) and the global coordinate systems
$\mathbf{u}$	hypervector of nodal displacements of the structure
$\mathbf{u}_c$	6-dimensional displacement vector of node c
$u_{c,n}$	nth component of displacement vector of node c (finite increment of $r_{c,n}$)
$( )^T$	transpose
$\bullet$	scalar product

1. INTRODUCTION

The aim of the paper is to survey concepts to analyse phenomena occurring during one-parameter (piecewise one-parameter) loading process of elastic bar structures composed of members rigidly connected to each other. Methods are particularly important that are suitable for numerical description of local and global loss of stability of bar structures supported along the boundary and covering large area. The structures in question can have many thousands of bars with high order of redundancy, and their strength and stability needs tremendous effort and computing time. Because of the large dimensions, the organization of computation is an important problem also from theoretical point of view.

For pin-jointed bar structures we have executed such an investigation, and we reported on the results in a series of papers (Szabó and Tarnai, 1993, 1997; Tarnai and Szabó, 1995). In order to reduce the computational needs we proposed a trihedral composition of the structure, and an efficient way of forming the inverse of the equilibrium matrix of the system worked out by Sherman and Morrison (1949).

The analysis of a bar structure with real (rigid) connections between bars is much more complicated than that with pin-jointed connections. The fundations of the procedure proposed in this paper for rigidly jointed bar structures have been laid down in the book by Szabó and Roller (1978), using the method of displacements; but the numerical handling in principle is the same as that we used for single-layer or double-layer pin-jointed space grids (Szabó and Tarnai, 1997). In order to keep the discussion simple all connections, even the connections to the foundation, will be supposed to be rigid.

In this paper, Section 2 presents the basic relationships for the change of state under small displacements that, apart from some little changes in the notation, are the same as in the book by Szabó and Roller (1978). In Section 3, the principle of the automatic generation of the basic equations and our procedure suggested for the calculation of the small-displacement change of state is shown, and our proposals for handling of local and global loss of stability are outlined.

The analysis of post-critical state and the presentation of our computational organization conception, necessary for calculation of space grids covering very large space, is left to a future paper.

2. BASIC RELATIONSHIPS

In the initial state, the structure contains bars of straight axis with constant cross-section. The bars rigidly join the nodes. The coordinates of the theoretical centres of the nodes are considered known in the initial state, the axis of a bar passes through the theoretical centres of the nodes, and in the initial state the length of a bar is determined by coordinates of the nodal points at the ends of the bar. During the change of state of the structure, the values of the coordinates of the nodes and the positions of the coordinate systems attached to the nodes change. It is suitable to give this change by a 6-dimensional vector.

Let us denote the initial coordinates and the displacement coordinates of node c according to the following:

$$\mathbf{r}_c^{\mathrm{T}} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 \end{bmatrix}_c = \begin{bmatrix} x & y & z & 0 & 0 & 0 \end{bmatrix}_c;$$
$$\mathbf{u}_c^{\mathrm{T}} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{bmatrix}_c = \begin{bmatrix} \delta_x & \delta_y & \delta_z & \varphi_x & \varphi_y & \varphi_z \end{bmatrix}_c.$$

After finite displacements, the new coordinates are contained in vector ($\mathbf{r}_c + \mathbf{u}_c$). The node coordinates of the structure are given in the x,y,z system. It is suitable that the data corresponding to a bar are given in a coordinate system attached to the bar, for instance, for bar i in the system $(\xi,\eta,\zeta)_i$ i = 1, 2,..., b. Axis ξ is identical to the centroidal, axis η is identical to the larger principal axis of inertia of the cross-section, and axis ζ is perpendicular to both axes ξ and η. If a vector **a** is given in the system $\Xi(\xi,\eta,\zeta)$, and it is denoted by **a**(Ξ), then this vector **a** in the system X(x,y,z) can be given by the transformation **a**(X) = **T** • **a**(Ξ) where $\mathbf{T} \bullet \mathbf{T}^{\mathrm{T}} = \mathbf{E}$ (unit matrix). The rotation matrix $\mathbf{T}_i$ of bar i, where e.g. sign couple x,ξ_i denotes the cosine of the angle between the axes x and ξ, is

$$\mathbf{T}_i = \begin{bmatrix} x,\xi & x,\eta & x,\zeta & & & \\ y,\xi & y,\eta & y,\zeta & & & \\ z,\xi & z,\eta & z,\zeta & & & \\ & & & x,\xi & x,\eta & x,\zeta \\ & & & y,\xi & y,\eta & y,\zeta \\ & & & z,\xi & z,\eta & z,\zeta \end{bmatrix}_i .$$

Internal forces in bar i of starting point h_i and end point k_i, associated with point k_i, are given by vector $\mathbf{s}_i$:

$$\mathbf{s}_i^{\mathrm{T}} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \end{bmatrix}_i = \begin{bmatrix} P_\xi & P_\eta & P_\zeta & M_\xi & M_\eta & M_\zeta \end{bmatrix}_i .$$

External forces can load the structure only at the nodes. Therefore, the force vector associated with the end point k_i, denoted by $\mathbf{s}_i$ instead of $\mathbf{s}_{ki}$, is sufficient to determine the internal forces at any cross-section of the bar. Internal forces in the cross-section at point h_i can be determined by the transfer matrix:

$$\mathbf{s}_{hi} = \mathbf{B}_i \bullet \mathbf{s}_i$$

where

$$\mathbf{B}_i = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & -l_i & & 1 & \\ & l_i & & & & 1 \end{bmatrix}$$

and the length of bar i is l_i. The components of the load vector at node c are concentrated forces R_x, R_y, R_z and couples N_x, N_y, N_z:

$$\mathbf{q}_c^{\mathrm{T}} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix}_c = \begin{bmatrix} R_x & R_y & R_z & N_x & N_y & N_z \end{bmatrix}_c .$$

If the displacement vector of the starting point h_i of bar i is $\mathbf{u}_{hi}(\Xi)$, then vector $\mathbf{u}_{ki}$ giving the rigid motion of end point k_i of bar i in its own coordinate system (Ξ_i) is

$$\mathbf{u}_{ki}(\Xi_i) = \mathbf{B}_i^{\mathrm{T}} \bullet \mathbf{u}_{hi}(\Xi_i).$$

Taking into account that $\mathbf{u}_{ki}(\mathrm{X}) = \mathbf{T}_i \bullet \mathbf{u}_{ki}(\Xi_i)$ and $\mathbf{u}_{hi}(\Xi_i) = \mathbf{T}_i^{\mathrm{T}} \bullet \mathbf{u}_{hi}(\mathrm{X})$, in the coordinate system (X):

$$\mathbf{u}_{ki}(\mathrm{X}) = \mathbf{T}_i \bullet \mathbf{B}_i^{\mathrm{T}} \bullet \mathbf{T}_i^{\mathrm{T}} \bullet \mathbf{u}_{hi}(\mathrm{X}).$$

Under the forces and couples applied at the end point k_i of bar i, which is considered rigidly built in at its starting point h_i, displacements arise at end point k_i. It is suitable to describe them in the coordinate system (Ξ_i). Doing so, the displacement at point k_i resulted from the force at k_i (that is, from force $\mathbf{s}_i$ in bar i) can be given by the expression

$$\mathbf{u}_{ki}(\Xi_i) = \mathbf{F}_i \bullet \mathbf{s}_i(\Xi_i)$$

where $\mathbf{F}_i$ is the flexibility matrix of bar i of length l_i and cross section with tensional, bending and twisting rigidities EA_i, $EJ_{\eta i}$, $EJ_{\zeta i}$ and $GJ_{\xi i}$ (but siplifying the notation and giving subscript i for the entire matrix instead of the entries of the matrix):

$$\mathbf{F}_i = \begin{bmatrix} l/EA & & & & & \\ & l^3/3EJ_\zeta & & & & l^2/2EJ_\zeta \\ & & l^3/3EJ_\eta & & -l^2/2EJ_\eta & \\ & & & l/GJ_\xi & & \\ & & -l^2/2EJ_\eta & & l/EJ_\eta & \\ & l^2/2EJ_\zeta & & & & l/EJ_\zeta \end{bmatrix}_i .$$

On the other hand, force $\mathbf{s}_i$ in bar i, that is, force in bar at point k_i, resulted from the displacement of the end point of the bar is obtained by the inverse of $\mathbf{F}$ as

$$\mathbf{s}_i(\Xi_i) = \mathbf{F}_i^{-1} \bullet \mathbf{u}_{ki}(\Xi_i).$$

2.1 EQUILIBRIUM EQUATIONS

Equilibrium of nodes working as finite rigid bodies, and equilibrium of elastic bars each connecting two nodes is in question. It is enough to write the relationships for an individual bar and for two nodes at the ends of the bar. Let h_i and k_i be two nodes connected by bar i whose starting point is h_i and end point is k_i. Force $\mathbf{s}_i$ in bar is applied by definition at end point k_i of the bar. On the surface of positive normal ξ of the cross-section h_i, at the same time at the node h_i, force $\mathbf{s}_{hi} = \mathbf{B}_i \bullet \mathbf{s}_i$ arises, and the bar i transmits force -$\mathbf{s}_i$ to node k_i. Equilibrium of bar i requires external loads $\mathbf{q}_{hi}$ and $\mathbf{q}_{ki}$ applied at nodes h_i and k_i, given in the coordinate system (X). Therefore, the equations of nodal equilibrium are

$$\begin{aligned} \mathbf{T}_i \bullet \mathbf{B}_i \bullet \mathbf{s}_i + \mathbf{q}_{hi} &= 0 \\ -\mathbf{T}_i \bullet \mathbf{s}_i + \mathbf{q}_{ki} &= 0. \end{aligned} \tag{1}$$

It is easy to check that bar i under forces -$\mathbf{s}_{hi}$ and $\mathbf{s}_i$ arising at the starting point and the end point is in equilibrium. From the elementary equilibrium equations (1), the equilibrium equation of a bar structure arbitrarily composed of rigidly connected bars:

$$\mathbf{G} \bullet \mathbf{s} + \mathbf{q} = 0. \tag{2}$$

The structure of the equilibrium equations is illustrated in the Appendix.

2.2 COMPATIBILITY EQUATIONS

It is supposed that, on bar i, *kinematical* load independent of the actual forces can be applied:

$$\mathbf{t}_i^{\mathrm{T}} = \begin{bmatrix} t_\xi & t_\eta & t_\zeta & \vartheta_\xi & \vartheta_\eta & \vartheta_\zeta \end{bmatrix}_i.$$

The relationship between displacement vectors $\mathbf{u}_{hi}$ and $\mathbf{u}_{ki}$ of the starting point and end point of bar i is described by the compatibility equation:

$$\mathbf{T}^{\mathrm{T}} \bullet \mathbf{u}_{ki} = \mathbf{B}_i^{\mathrm{T}} \bullet \mathbf{T}_i^{\mathrm{T}} \bullet \mathbf{u}_{hi} + \mathbf{F}_i \bullet \mathbf{s}_i + \mathbf{t}_i$$

or after rearrangement:

$$\mathbf{B}_i^{\mathrm{T}} \bullet \mathbf{T}_i^{\mathrm{T}} \bullet \mathbf{u}_{hi} - \mathbf{T}_i^{\mathrm{T}} \bullet \mathbf{u}_{ki} + \mathbf{F}_i \bullet \mathbf{s}_i + \mathbf{t}_i = 0. \tag{3}$$

From the elementary compatibility equations (3), for all displacement vectors we have the compatibility equation of the structure:

$$\mathbf{G}^{\mathrm{T}} \bullet \mathbf{u} + \mathbf{F} \bullet \mathbf{s} + \mathbf{t} = 0 \tag{4}$$

where $\mathbf{G}^{\mathrm{T}}$ is the compatibility matrix.

3. AUTOMATIC GENERATION OF THE FUNDAMENTAL EQUATIONS, AND THE CALCULATION OF SMALL DISPLACEMENT CHANGE OF STATE

Under given structural geometry, the generation of nodes and the initial coordinates can be executed with the procedure worked for bar-and-joint assemblies (Szabó and Tarnai 1997). For composition of the list of bars, the trihedral generation can be used. The list RU(i, h_i, k_i) of bars constitutes the base of the generation of equilibrium matrix **G**.

3.1 EQUATION OF CHANGE OF STATE FOR SMALL DISPLACEMENTS

If we unite the equilibrium and the compatibility equations in a matrix form, then we have

$$\begin{bmatrix} & \mathbf{G} \\ \mathbf{G}^{\mathrm{T}} & \mathbf{F} \end{bmatrix} \bullet \begin{bmatrix} \mathbf{u} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{q} \\ \mathbf{t} \end{bmatrix} = 0$$

that is called the equation of change of state. Considering it as a set of two equations:

$$\mathbf{G} \bullet \mathbf{s} + \mathbf{q} = 0$$
$$\mathbf{G}^{\mathrm{T}} \bullet \mathbf{u} + \mathbf{F} \bullet \mathbf{s} + \mathbf{t} = 0$$

and supposing that det(**F**) $\neq 0$, it is easy to give the main points of the solution procedure.

(a) We express force vector **s** from the second equation:

$$\mathbf{s} = -\mathbf{F}^{-1} \bullet \mathbf{G}^{\mathrm{T}} \bullet \mathbf{u} - \mathbf{F}^{-1} \bullet \mathbf{t}.$$

(b) By substituting it in the first equation we have

$$-\mathbf{G} \bullet \mathbf{F}^{-1} \bullet \mathbf{G}^{\mathrm{T}} \bullet \mathbf{u} - \mathbf{G} \bullet \mathbf{F}^{-1} \bullet \mathbf{t} + \mathbf{q} = 0.$$

(c) Introducing the notation $\mathbf{K} = \mathbf{G} \bullet \mathbf{F}^{-1} \bullet \mathbf{G}^{\mathrm{T}}$ it can be written in the form

$$\mathbf{K} \bullet \mathbf{u} = \mathbf{q} - \mathbf{G} \bullet \mathbf{F}^{-1} \bullet \mathbf{t} \tag{5}$$

whence **u** can be determined by inverting **K**.
(d) Knowing **u** and using the expression in (a), **s** can be determined.

3.2 GENERATION OF MATRIX G

The number of block-rows of **G** is equal to the number of the internal joints j, and the number of block-columns of **G** is equal to the number of bars b. In the block-column corresponding to the bar i: in the block-row corresponding to the starting point h_i of the bar, matrix $\mathbf{T}_i \bullet \mathbf{B}_i$ is present, in the block-row corresponding to the end point k_i of the bar, if $k_i \le j$, $-\mathbf{T}_i$ is present, and 0 is otherwise.

3.3 GENERATION OF STIFFNESS MATRIX K

For generation of stiffness matrix **K**, the following algorithm is used instead of that used in Szabó and Roller (1978). This consists of a simple expression:

$$\mathbf{K} = \sum_{i=1}^{b} \mathbf{g}_i \bullet \mathbf{F}_i^{-1} \bullet \mathbf{g}_i^{\mathrm{T}}$$

where $\mathbf{g}_i$ is the ith block-column in matrix **G**:

$$\mathbf{G} = [\mathbf{g}_1 \quad \mathbf{g}_2 \quad \dots \quad \mathbf{g}_b].$$

3.4 THE PROPOSED PROCEDURE FOR CALCULATION OF SMALL DISPLACEMENT CHANGE OF STATE

It is supposed that in (5) $\mathbf{t} = 0$. It is so, because either there is no kinematic load on the structure or the kinematic load is reduced to a nodal load by the expression $\mathbf{q}_{red} = -\mathbf{G} \bullet \mathbf{F}^{-1} \bullet \mathbf{t}$, and $\mathbf{q} - \mathbf{q}_{red}$ is considered as **q**. Then the equation

$$\mathbf{K} \bullet \mathbf{u} = \mathbf{q}$$

should be solved with a suitable procedure, or taking the inverse $\mathbf{K}^{-1}$, the displacement coordinates can be determined for arbitrary **q**:

$$\mathbf{u} = \mathbf{K}^{-1} \bullet \mathbf{q}.$$

At the beginning of the small displacement procedure, for the increase of the one-parameter load, the change of **u** is accepted to be linear up to a limit where the crookedness of compressed bars exceeds a permissible (prescribed) value. Then, the change of state is still considered as that of small displacement, but the structure does not consist of bars of straight axis any longer but curved axis. According to this fact, the flexibility matrix should be modified, as the force component parallel with axis ξ, acting at the end of the bar, produces a bending moment in the bar. The procedure presented below is based on this phenomenon, and preserves the properties of the second-order theory.

(a) In each load step, forces $\mathbf{s}_i$ in bars and displacements **u** of nodes become known. Each component of a force in bar produces deformations of the bar considered as a cantilever rigidly built in at the starting point h_i, and all these deformations between h_i and k_i provide the shape of the curved bar. We take its projections in the planes ξ,η and ξ,ζ and. The bars are loaded only at their ends, so the projection of a curved bar can be only a cubic curve. Therefore, in the procedure, the curve is given as a polynomial of degree three, and the coefficients of the terms in the polynomial are determined from the boundary conditions according to the fitting at the starting and end points. For displacement coordinates $\mathfrak{z}$, $\mathfrak{y}$, φ_η, φ_ζ, the curves generated in this way are shown in Fig. 1.

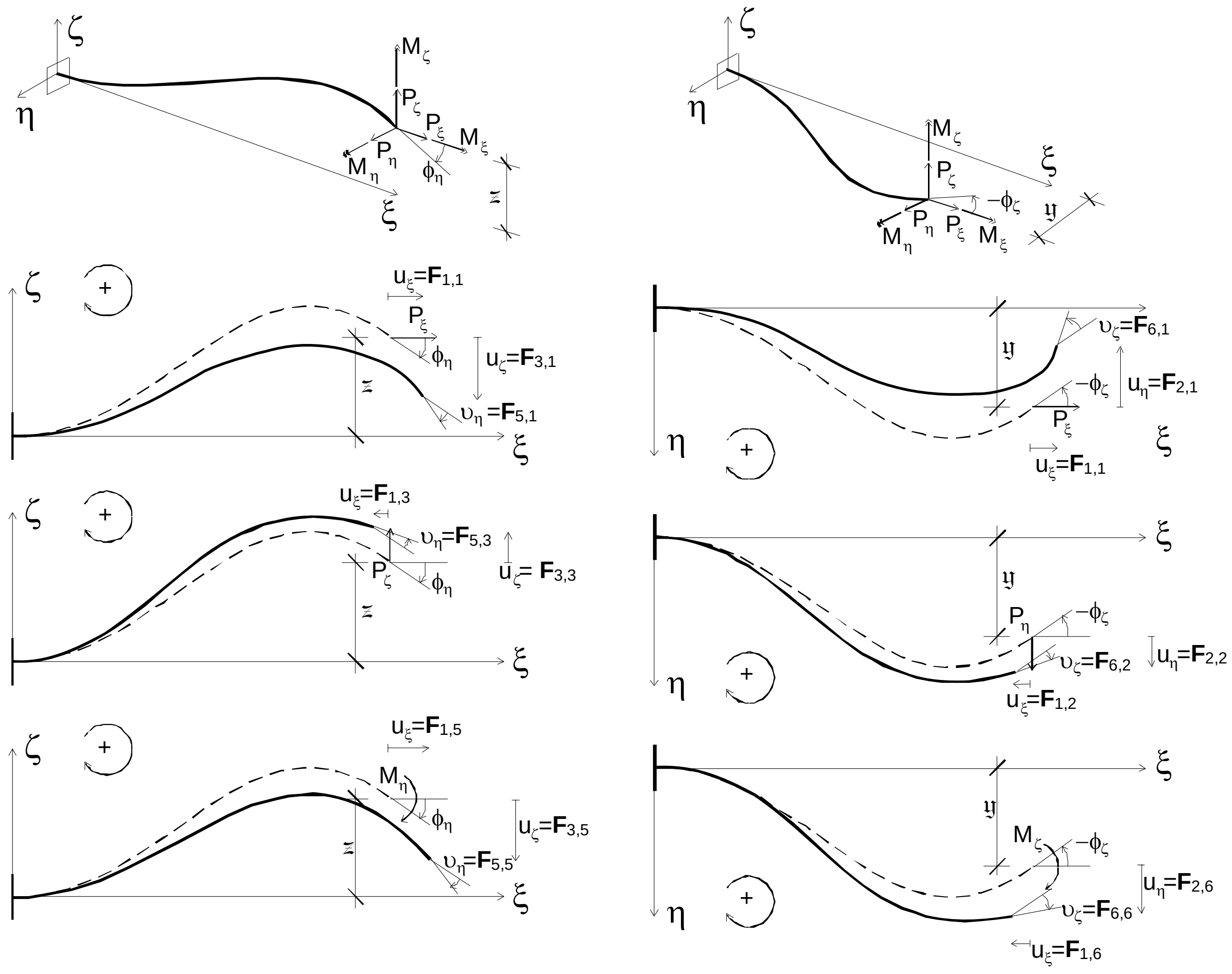

Figure 1

(b) With the help of displacement coordinates and force components arising in the preceding load step, displacement coordinates for unit force components at the end of the bar can be determined, and with these a new flexibility matrix $\mathbf{F}_i$ is produced. The general form of the new flexibility matrix is

$$\mathbf{F}=\begin{bmatrix} F_{11} & F_{12} & F_{13} & 0 & F_{15} & F_{16} \\ F_{21} & F_{22} & 0 & F_{24} & 0 & F_{26} \\ F_{31} & 0 & F_{33} & F_{34} & F_{35} & 0 \\ 0 & F_{42} & F_{43} & F_{44} & 0 & 0 \\ F_{51} & 0 & F_{53} & 0 & F_{55} & 0 \\ F_{61} & F_{62} & 0 & 0 & 0 & F_{66} \end{bmatrix}$$

and the elements of $\mathbf{F}$ are

$$F_{11}=\frac{\frac{13}{35}z^2 l-\frac{13}{210}z\varphi_\eta l^2+\frac{1}{105}\varphi_\eta^2 l^3}{EJ_\eta}+\frac{l}{EA}+\frac{\frac{13}{35}y^2 l+\frac{13}{210}y\varphi_\zeta l^2+\frac{1}{105}\varphi_\zeta^2 l^3}{EJ_\zeta}$$

$$F_{21}=F_{12}=\frac{-\frac{7}{20}y\,l^2-\frac{1}{30}\varphi_\zeta l^3}{EJ_\zeta} \qquad F_{22}=\frac{l^3}{3EJ_\zeta}$$

$$F_{31}=F_{13}=\frac{-\frac{7}{20}z\,l^2+\frac{1}{30}\varphi_\eta l^3}{EJ_\eta} \qquad F_{33}=\frac{l^3}{3EJ_\eta}$$

$$F_{42}=F_{24}=\frac{-\frac{1}{2}z\,l+\frac{1}{12}\varphi_\eta l^2}{GJ_\xi} \qquad F_{43}=F_{34}=\frac{\frac{1}{2}y\,l+\frac{1}{12}\varphi_\zeta l^2}{GJ_\xi} \qquad F_{44}=\frac{l}{GJ_\xi}$$

$$F_{51}=F_{15}=\frac{\frac{1}{2}z\,l-\frac{1}{12}\varphi_\eta l^2}{EJ_\eta} \qquad F_{53}=F_{35}=-\frac{l^2}{2EJ_\eta} \qquad F_{55}=\frac{l}{EJ_\eta}$$

$$F_{61}=F_{16}=\frac{-\frac{1}{2}y\,l-\frac{1}{12}\varphi_\zeta l^2}{EJ_\zeta} \qquad F_{62}=F_{26}=\frac{l^2}{2EJ_\zeta} \qquad F_{66}=\frac{l}{EJ_\zeta}$$

According to the preceding, the transfer matrix $\mathbf{B}$ also changes to

$$\mathbf{B}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -z & y & 1 & 0 & 0 \\ z & 0 & -l & 0 & 1 & 0 \\ -y & l & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since before and after a load step, the coordinate systems ξ, η, ζ attached to the starting point of the bar are not identical to each other, a basis transformation (Rózsa, 1974) should be applied.
(c) In the different load steps, the commands for the generation of matrices **F** and **B** are unchanged, but in an actual step they must apply the values of **u**, **s** obtained in the preceding step.

3.5 CRITICAL STATES RECOGNIZABLE IN THE SMALL DISPLACEMENT CHANGES OF STATE

Using small displacement theory, Timoshenko and Gere (1961) have called the compressive force on an elastic crooked bar *critical*, if that force results in infinite deflection of the bar. Analogously to this asymptotic definition, we call the load parameter globally critical if the load causes infinite displacements of the elastic structure (with bars crooked under the increasing load). In case of plasticity is taken into account, we consider a state to be "locally critical" (not in the sense of the theory of stability), if in one (or more) bar(s), a plastic hinge appears, and that (those) bar(s) cannot be loaded more. In the procedure, such a state is modeled that the forces transferred from the bar to the node are considered approximately constant, but the submatrices corresponding to such bar(s) are removed from matrix **K**.

Then the procedure is continued with the modified matrix **K**, according to Subsection 3.4.

4. CONCLUSIONS

Using the data of a simple double-layer pin-jointed space grid analysed by Szabó and Tarnai (1997), we executed a pilot numerical investigation of the same grid but with rigid nodes, for one-parameter load such that only the nodes of the upper layer were loaded. The main conclusions are as follows.
(a) It is useful to increase significantly the stiffness of the supporting bars along the boundary in order to avoid the possibility that the decrease in the local load carrying capacity starts at these bars.
(b) The maximum stress reached the yield limit first in the same bars as in which, in the pin-jointed structure, the normal force first reached the critical value. However, the load level for the first yield in the rigidly jointed structure was 2.6 times higher than that for first buckling in the pin-jointed structure.

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REFERENCES

Rózsa, P. (1974). *Linear algebra and its applications* (in Hungarian). Műszaki Könyvkiadó, Budapest.

Sherman, J. and Morrison, W.J. (1949). Adjustment of an inverse matrix corresponding to changes in a given row of the original matrix. *The Annals of Math. Statistics* **21**, 124.

Szabó, J. and Roller, B. (1978). *Anwendung der Matrizenrechnung auf Stabwerke*. Akadémiai Kiadó, Budapest.

Szabó, J. and Tarnai, T. (1993). General theory and numerical analysis of single- and double-layer pin-jointed space grids. *Space Structures 4* (Ed.: G.A.R. Parke and C.M. Howard), Thomas Telford, London, Vol. **1**, pp. 715-722.

Szabó, J. and Tarnai, T. (1997). Analysis of critical and postcritical states of pin-jointed double-layer space grids fitted to surfaces of double curvature. *Proc. of the Int. Coll. on Computation of Shell and Spatial Structures* (Ed.: Y.B. Yang), Taipei, pp. 121-126.
Tarnai, T. and Szabó, J. (1995). Numerical description of the complete equilibrium path of automatically generated space frames. *Spatial Structures: Heritage, Present and Future. Proc. IASS Symp.* (Ed.: G.C. Giuliani), Milano, Vol. **1**, pp. 669-676.
Timoshenko, S.P. and Gere, J.M. (1961). *Theory of elastic stability*. McGraw-Hill, New York.

APPENDIX

To illustrate the relationship between the systems of equations (1) and (2) we show a simple structure (Fig. 2). To each of the bars denoted by 1 to 8 there corresponds a pair of equilibrium equations

$$\mathbf{T}_i \bullet \mathbf{B}_i \bullet \mathbf{s}_i + \mathbf{q}_{hi} = 0$$
$$-\mathbf{T}_i \bullet \mathbf{s}_i + \mathbf{q}_{ki} = 0$$

from which the system of equilibrium equations of the structure in Fig. 2 is composed:

$$\begin{bmatrix} \mathbf{T}_1 \bullet \mathbf{B}_1 & \mathbf{T}_2 \bullet \mathbf{B}_2 & \mathbf{T}_3 \bullet \mathbf{B}_3 & & & & & \\ -\mathbf{T}_1 & & & \mathbf{T}_4 \bullet \mathbf{B}_4 & \mathbf{T}_5 \bullet \mathbf{B}_5 & & & \\ & -\mathbf{T}_2 & & & & \mathbf{T}_6 \bullet \mathbf{B}_6 & \mathbf{T}_7 \bullet \mathbf{B}_7 & \\ & & & -\mathbf{T}_4 & & -\mathbf{T}_6 & & \mathbf{T}_8 \bullet \mathbf{B}_8 \end{bmatrix} \bullet \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \\ \mathbf{s}_4 \\ \mathbf{s}_5 \\ \mathbf{s}_6 \\ \mathbf{s}_7 \\ \mathbf{s}_8 \end{bmatrix} + \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \\ \mathbf{q}_4 \end{bmatrix} = 0.$$

Figure 2