Lightweight Structural Damage Assessment by Static Defect Energy

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ABSTRACT

Some global damage detection parameters have been proposed by using of system dynamic responses such as the natural frequencies, frequency response functions, mode shapes and energies, etc., in the past few years. The objective of this paper is to propose a new parameter, termed as the static defect energy, by applying the perturbation theory to the oscillatory elastic solid and derive a formula that can be obtained under the static load situation. This candidate parameter possesses the ability to detect, locate and quantify structural damage. A lightweight member made of two channels connected back to back is selected as a numerical example to confirm the correctness of this parameter. It is shown that under static load, two energy levels are separated by a vertical step right at the damaged location. Furthermore, it is interesting to find that the height of vertical step is proportional to the severity of damage.

1. BACKGROUND

Lightweight structure has become one of the major structural types in sports, leisure and entertainment facilities. It provides the following advantages in material aspect as well as in building construction: high strength-to-weight ratios, mass production and easy prefabrication, fast erection and installation, nonshrinking and noncreeping at ambient temperatures. Its unusual cross-sections designed for any specific purpose can be economically produced by cold-forming operations. Load-carrying panels not only can provide surfaces for roof, floor and wall, but they can also act as shear diaphragms to resist in-plane shear forces and to prevent sidesway if properly connected. Although cold-formed steel has been mainly used in car bodies, railway coaches and storage racks, its applications to large-scale structures as primary members has become more and more frequently used and important.

For large-scale public structures, safety has been always the most important issue to be concerned. Therefore, efforts has to be done to assure the integrity of a structure. A long term structural monitoring system is required for this purpose. Stress concentration at the damaged area has been recognized as one of the main factors contributing to the collapse of structures. Before any repair works can be done to prevent the disasters, locations of damage have to be determined in the first stage. Sometimes, owing to the budget or some other reasons, repair works can not be proceeded immediately even if the damage locations were known. Decisions on the priority of the repairing works then depend on the severity of the damage. To have a full understanding and control over the problems, information about presence of damage in the early stage, precise locations of the damage, and it's severity becomes very important.

Traditional nondestructive evaluation methods can be employed to detect damage of various types for various materials. But generally speaking, they are more suitable for mechanical structures than for civil structures because of their localized nature of operation. For large-scale civil structures, global damage detection methods are more desired. They can be approximately classified, according to the use of the dynamic characteristics, into four categories: (1) using dynamic response information of system, such as, compliance, mobility and inertance, etc. [1-5] (2) using information of eigenvalues, such as, frequency variation, eigenvalue ratio, etc. [6-8] (3) using information of mode shapes, such as, displacement, rotation, curvature and strain mode shapes, etc. [9-14] (4) using energy-related parameters, such as, cumulative dissipated energy [15], strain energy density [16], seismic-energy dissipation [17], and defect energy [18-20].

There are also some common limitations in the dynamic measurement, for example, the signal-noise ratio, the uncertainty of the random vibration, accuracy of the modal testing, and energy required for the shaker or impact hammer. In addition, it takes specially trained technician to operate the expensive equipment. To perform a static experiment is usually much easier than a dynamic one.

2. FORMULATION

The static defect energy (SDE) formulation is originated from the extended study of the (dynamic) defect energy (DDE) which can be referred to some references [18-20]. Equations related to DDE will be listed and kept as simple as a reasonable complete treatment of the this part allows. For the spatial continuous solid shown in Fig. 1, we assume it to be linear, elastic and its deformation very small under external loading. The governing equations of an oscillatory system for the n^{th} eigenmode can be expressed as:

$$\sigma^n_{ij,j} + \rho\omega^2_n u^n_i = 0 \qquad \text{in} \qquad D^n \tag{1a}$$

$$\overline{T^n_i} = \sigma_{ij} n_j \qquad \text{on} \qquad C^n_T \tag{1b}$$

$$\overline{u^n} = u_i \qquad \text{on} \qquad C^n_u \tag{1c}$$

where super- and subscripts "n" is used to represent parameters of the n^{th} eigenmode of the homogeneous material to distinguish from the same notations without "n" of the non-homogeneous material. σ_{ij} is the stress tensor, ρ is the mass per unit volume, ω is the angular eigenfrequency, u_i is the displacement vector, T_i is the surface traction, n_i is the unit outward normal vector on the boundary C_T. C_T is part of the boundary where the traction are zero and C_u is part of the boundary where the displacements are zero. D is the body domain. Superscript "bar" is used as an indication of the prescribed boundary condition. A comma in the subscript means the derivative of that quantity with respect to a spatial coordinate. Let x_j be the spatial coordinate, then

$$\sigma^n_{ij,j} = \frac{\partial \sigma^n_{ij}}{\partial x_j} \tag{2}$$

It is well known in fracture mechanics that the J integral has been related to potential energy release rate associated with cracks in linear or nonlinear materials. Its integration path must be a closed loop. If there exist any crack within the explicit integration paths, J is not equal to zero. On the other hand, if there is no crack or material non-homogeneity within the paths, J is equal to zero. Assume that the body forces were neglected and Young‘s modulus is a constant. Similar to the J integral defined in fracture mechanics, the rate of energy release per unit of crack extension vector, F_i, can be such defined as to satisfy the governing equations:

$$F_i = \int_A \left[(W - T)\delta_{ij} - \sigma_{kj} \frac{\partial u_k}{\partial x_j} \right] n_j dA \tag{3}$$

in which A is a closed integration curve, W is the total strain energy density per unit volume, T is the kinetic energy per unit volume, and δ_{ij} is the Kronecker delta. The function of vector F_i is similar to the J integral. By changing path of integration, the existence of the non-homogeneous material can be found if F_i value is non-zero. By narrowing down the path of integration, location of the non-homogeneous material can be determined precisely. If Young’s modulus of this non-homogeneous area is set to be zero for a special case, it is the situation of cracking. For any member of a 2-D beam/frame structure shown in Fig. 2, the vector F_i can be reduced to a scalar

$$F = \left[-(W + V\theta + T) \right]_{x_1}^{x_2} \tag{4}$$

where V is the shear force and θ is the rotation. The integration area of vector F_i in a 3-D case can now be reduced to the evaluation at two points, x_1 and x_2 only. The kinetic energy T, and the total strain energy density per unit volume W, respectively are

$$T = \frac{1}{2}\omega_n^2 \rho A \left(u^2 + \frac{I}{A}\theta^2 \right) \tag{5}$$

$$W = W(\kappa) + W(\gamma) \tag{6}$$

Suppose P is an energy related quantity, whose contributions are energy terms of F evaluated at a single point x_1 or x_2 only. Then, it’s complete form would be

$$P = W(\kappa) + W(\gamma) + V\theta + T \tag{7}$$

For a homogeneous material, $F_i = 0$, that is

$$[P]_{x_1} = [P]_{x_2} \tag{8}$$

While for a non-homogeneous material, $F_i \neq 0$, that is

$$[P]_{x_1} \neq [P]_{x_2} \tag{9}$$

P is now a scalar quantity. Plot of P against location of a member would appear two different energy constants separated by a vertical step right at the damaged location as shown in Fig. 3.

Since changes in the geometry is very small, perturbation theory can be applied. The first application of perturbation theory to changes of geometry in eigenvalue problems was made by Brillouin in 1937. Several papers have been published on this subject since then. Most of these applications in this theory were used to predict the eigenvalue-changed problems. But the purpose of using it here is to get rid of the inertia force by the first order approximation and to simplify the calculations. Assume structural conditions were exactly the same as stated by Gudmudson [21]. For a linear elastic structure with no surface traction, the intact eigenvalues and eigenvectors are solutions to the problem

$$\sigma_{ij,j}^n + \rho\omega_n^2 u_i^n = 0 \quad \text{in } D^n \tag{10a}$$

$$T_i^n = 0 \quad \text{on } C_T^n \tag{10b}$$

$$u_i^n = 0 \quad \text{on } C_u^n \tag{10c}$$

and are assumed to be known. Consider changes of geometry which introduce new traction-free boundaries. The disturbed eigenvalue problem takes the following form:

$$\sigma_{ij,j} + \rho\omega^2 u_i = 0 \qquad \text{in } D \tag{11a}$$

$$T_i = 0 \qquad \text{on } C_T \tag{11b}$$

$$u_i = 0 \qquad \text{on } C_u \tag{11c}$$

Suppose that the shift in the n^{th} resonance frequency is of primary interest. The n^{th} disturbed eigenmode can be written as:

$$u_i = u_i^n + \Delta u_i \tag{12}$$

where Δu_i is a correction to u_i^n . By combining Eq. (10) to (12), the equations for the correction, Δu_i , are obtained.

$$\rho\left(\omega^2 - \omega_n^2\right)u_i^n + \Delta\sigma_{ij,j} + \rho\omega^2\Delta u_i = 0 \qquad \text{in } D \tag{13a}$$

$$\Delta T_i = -T_i^n \qquad \text{on } C_T \tag{13b}$$

$$\Delta u_i = 0 \qquad \text{on } C_u \tag{13c}$$

All the "Δs" represent the correction quantities. Assume that the size of the undisturbed body is of order "L" and the cut-out is of order "a" and that

$$\frac{a}{L} << 1 \tag{14}$$

If the undisturbed eigenmode, u_i^n , is of the order "L", the correction, Δu_i, can be assumed to be of order "a." A typical distance close to the cut-out is "a"; hence the correction is assumed to vary over a distance "a." The following dimensionless variables specified with superscript "*" can now be introduced:

$$\Delta u_i = a\Delta u_i^*\left(\frac{x_i}{a}\right) \tag{15a}$$

$$\Delta\sigma_{ij} = E\Delta\sigma_{ij}^*\left(\frac{x_i}{a}\right) \tag{15b}$$

$$u_i^n = Lu_i^{*n}\left(\frac{x_i}{L}\right) \tag{15c}$$

$$\sigma_{ij}^n = E\sigma_{ij}^{*n}\left(\frac{x_i}{L}\right) \tag{15d}$$

$$\omega_n^2 = \frac{E}{\rho}\frac{1}{L^2}\omega^{*2} \tag{15e}$$

$$\omega^2 = \frac{E}{\rho}\frac{1}{L^2}\omega^{*2}(1+\delta) \tag{15f}$$

in which x_i is a position vector of an interested point and δ is the shift quantity. Introducing Eq. (15) to Eq. (13) and neglecting the second and higher order terms, the first order approximation to Δu_i can be calculated as:

$$\Delta\sigma_{ij,j} = 0 \qquad \text{in } D \tag{16a}$$

$$\Delta T_i = -T_i^n \qquad \text{on } C_T \tag{16b}$$

$$\Delta u_i = 0 \qquad \text{on } C_u \tag{16c}$$

Result from Eq. (16) indicates that the inertia force of the disturbed structure has been removed. Therefore, energy scalar P can be reduced to

$$P = \frac{1}{2}EI\kappa^2 + \frac{1}{2}GA\gamma^2 + V\theta \tag{17}$$

The calculation procedures will be reduced greatly since the kinetic energy T, which is a second order term, has been discarded. What more important is that it has become a static problem. Ingredients needed for P and F can be obtained directly from static method.

The idea of using F for damage detection is to show the non-zero quantities across the damaged locations. An alternative way of presenting it is to plot the energy scalar terms P along the member. For numerical simulation in dynamic problems, since eigenvalues are obtained from numerical method instead of from the analytical solution, errors are inevitable especially when dealing with very small changes in the stiffness matrices. It was found that the accuracy of the calculation results is sensitive to the applied finite element code itself as well as the post-processing procedures. It is always possible to obtain an oscillating result even for a homogeneous material if the FEM code is not good enough. Actually, the deflected shape of a beam under static load is similar to the first mode under vibration. The static situation can be treated as a special case of the dynamic analysis.

Similar to the errors that occur in the numerical simulations, they can also be found in the experimental measurements. Good results can not be obtained without filtering the errors. Either from numerical simulation or from the field implementation point of view, using only one set of P value for damage evaluation may not obtain very clear information as it is expected from the theory. Therefore, it is suggested here to take the differences of P between two states: at a reference time "t" and after a duration "dt". Let U_{ds} be the difference of P between the two stages.

$$U_{ds} = \{P\}_t - \{P\}_{t+dt} \tag{18}$$

By plotting the magnitude of U_{ds} along the structure, a sharp vertical step can be seen right at the damaged location. Since this quantity is used to evaluate change of energy induced by the structural/material deficiencies by applying static load, it is called the static defect energy (SDE) to distinguish from the dynamic defect energy (DDE) proposed previously. The most important message from the static defect energy parameter is that it is comprised of all the energy quantities from the following ingredients: curvature κ, shear strain γ, shear force V, and rotation θ, rather than the application of each individual term itself. All the terms can be obtained only by static analysis and/or experiment.

3. NUMERICAL EXAMPLE

A flexural member with cold-formed steel double-channels cross section is selected as an numerical example. This shape is made of SS400 steel according to the CNS specification. It is simply supported and laterally braced adequately in the compression flange. The geometric shape, dimensions, properties and finite element model are illustrated in Fig. 4. Three damage cases which are 10, 30 and 50% severity, will be imposed on element 13. 10% damage means a 10% deduction of Young's modulus in material in the FEM model while other properties remain unchanged. In order to create a deformed shape, a 2.94 KN static concentrated transverse load is applied at node 7. The designed load should be heavy enough to produce measurable deformation but not to exceed the allowable maximum deflection which is specified in the codes. In addition, loading magnitude and position for both intact and damage structures should be exactly the same. For noncompact thin-walled sections, the webs of beams should also be checked for shear and bearing crippling.

From finite element analysis, results of the curvature, shear force, shear strain, rotation, displacement and moment of the intact beam at each single element are listed in Table 1. Substitute these numbers into Eq. (17), P term and its ingredients can be calculated and plotted in Fig. 5. It shows that the P quantity is a constant everywhere except when it is across node 7 which is the loading point. In the assumption, surface traction is not allowed to exist between two evaluation points. Therefore, a $q\theta$ term can be added to each element after node 7 to modify P. "q" is the magnitude of the static load. However, the modification procedures can also be ignored because the load application points is always known.

For damaged structures, calculation procedures for stress-strain field and energy ingredients are exactly the same as before. By substituting two P terms into Eq. (18), the "raw" SDE is obtained. For demonstration, the smallest raw SDE quantity can be set as a datum. It can be either a positive or a negative unity. The rest of the raw quantities are re-calculated by taking the ratio. Figure of the SDE Ratio for the 10% damage case is shown in Fig. (6). This is also the typical demonstration appears for all the other examples examined. When three sets of the unnormalized raw SDE data were plotted together as shown in Fig. (7), it can be found that vertical height of the energy level is proportional to damage severity. These characteristics have also been confirmed in all the other simulation cases for different structures.

4. CONCLUSIONS

If the dynamic experiment is hard to achieve, a static one is preferred. By using the SDE alone, damage information of lightweight structures can be obtained and assessed. In addition, there are some advantages about the SDE parameter: (1) It is a simple, stable and reliable damage detection parameter. Only four quantities are required to be measured, not any one of them is needed from the damaged location. (2) All the simulation results demonstrate good and steady characteristics for lightweight structures. (3) It provides better sensitivity to a localized damage. (4) Calculation is very simple, no integration process is involved. It can be done simply by an ordinary calculator. (5) Only a few measuring stations are needed to obtain damage information. (6) Load can be applied at any convenient locations with arbitrary magnitude as long as it can produce measurable strain and displacement quantities. (7) It is feasible to establish long term safety monitoring system by using SDE.

ACKNOWLEDGMENT

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Table 1 Deflected Quantities of the Intact Structure

Elem. No.	Curvature	Shear strain	Shear force	Rotation	Displacement	Moment
1	6.21E-06	-2.50E-05	-2.10E+02	-8.39E-03	-8.41E-02	2.10E+03
2	1.86E-05	-2.50E-05	-2.10E+02	-8.14E-03	-2.50E-01	6.30E+03
3	3.11E-05	-2.50E-05	-2.10E+02	-7.64E-03	-4.08E-01	1.05E+04
4	4.35E-05	-2.50E-05	-2.10E+02	-6.89E-03	-5.54E-01	1.47E+04
5	5.59E-05	-2.50E-05	-2.10E+02	-5.90E-03	-6.82E-01	1.89E+04
6	6.83E-05	-2.50E-05	-2.10E+02	-4.66E-03	-7.88E-01	2.31E+04
7	7.19E-05	1.07E-05	9.00E+01	-3.26E-03	-8.67E-01	2.43E+04
8	6.65E-05	1.07E-05	9.00E+01	-1.87E-03	-9.18E-01	2.25E+04
9	6.12E-05	1.07E-05	9.00E+01	-5.95E-04	-9.43E-01	2.07E+04
10	5.59E-05	1.07E-05	9.00E+01	5.77E-04	-9.43E-01	1.89E+04
11	5.06E-05	1.07E-05	9.00E+01	1.64E-03	-9.21E-01	1.71E+04
12	4.53E-05	1.07E-05	9.00E+01	2.60E-03	-8.78E-01	1.53E+04
13	3.99E-05	1.07E-05	9.00E+01	3.45E-03	-8.17E-01	1.35E+04
14	3.46E-05	1.07E-05	9.00E+01	4.20E-03	-7.41E-01	1.17E+04
15	2.93E-05	1.07E-05	9.00E+01	4.84E-03	-6.50E-01	9.90E+03
16	2.40E-05	1.07E-05	9.00E+01	5.37E-03	-5.48E-01	8.10E+03
17	1.86E-05	1.07E-05	9.00E+01	5.79E-03	-4.36E-01	6.30E+03
18	1.33E-05	1.07E-05	9.00E+01	6.11E-03	-3.17E-01	4.50E+03
19	7.99E-06	1.07E-05	9.00E+01	6.33E-03	-1.92E-01	2.70E+03
20	2.66E-06	1.07E-05	9.00E+01	6.43E-03	-6.44E-02	9.00E+02

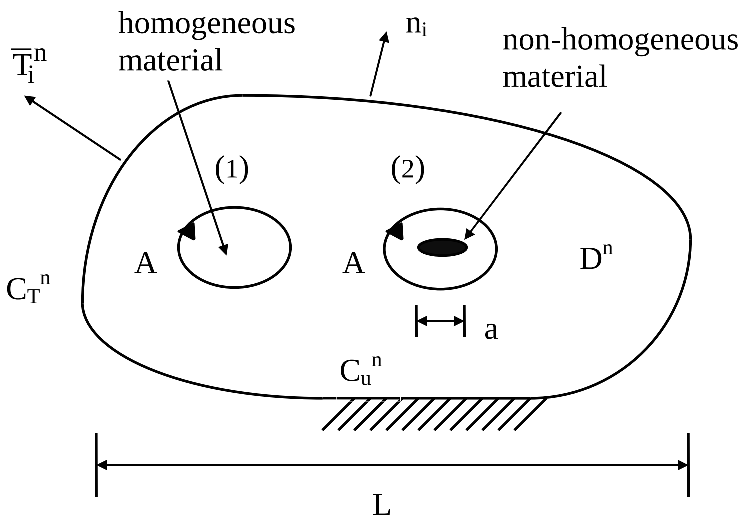


Fig. 1 Elastic Continuous Solid with the Explicit Integral Loop Over a (1) Homogeneous (2) Non-homogeneous area

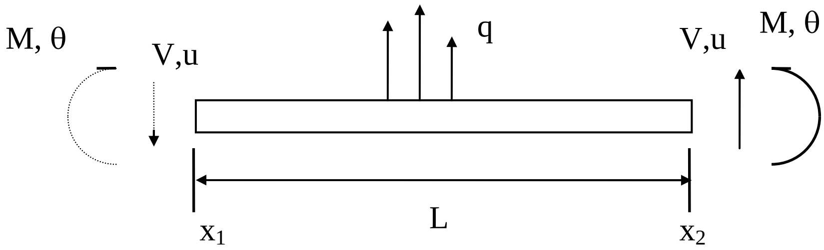


Fig. 2   A 2-D Beam/Frame Member and Its Sign Convention

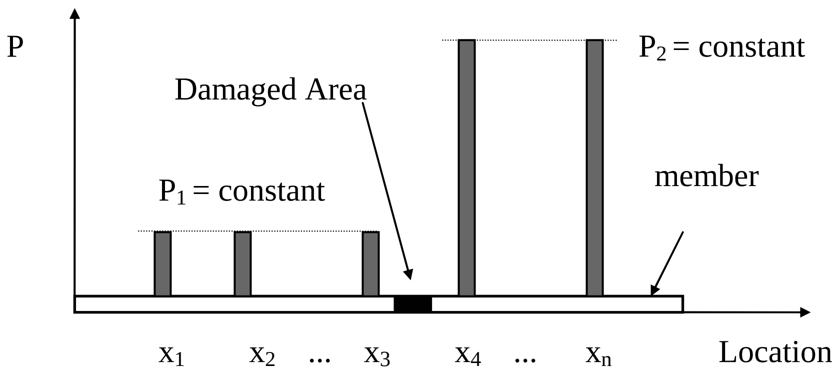
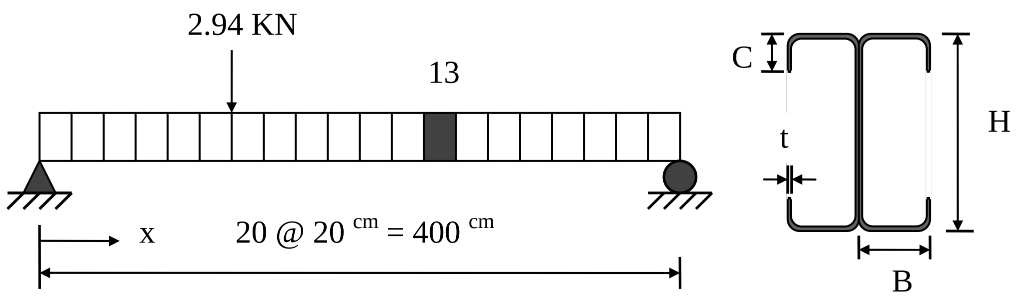


Fig. 3   Quantity of P at the Evaluation Points



Lightewight cold-formed steel member:

unit weight = 7860 kg/m^3
FEM mesh : 21 nodes, 20 cm per element
10, 30 and 50% damage at element 13

2 channels connected back to back
H×B×C×t
=100×50×20×2.3 ^{mm}

Fig. 4   The Steel Beam Dimensions, Properties and FEM Model

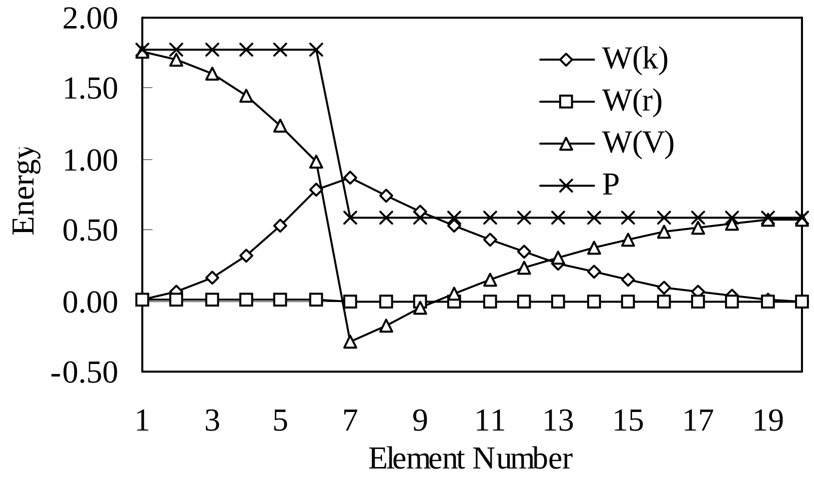


Fig. 5 Ingredients of the SDE for Inatct Structure

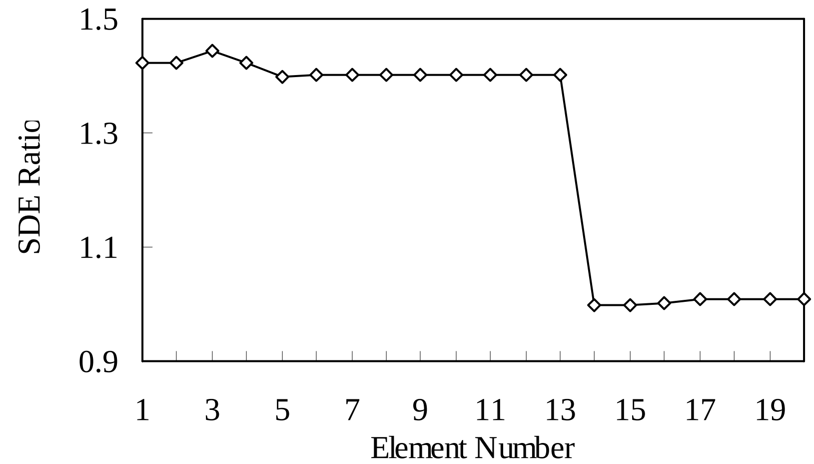


Fig. 6 SDE Ratio for 10% Damage

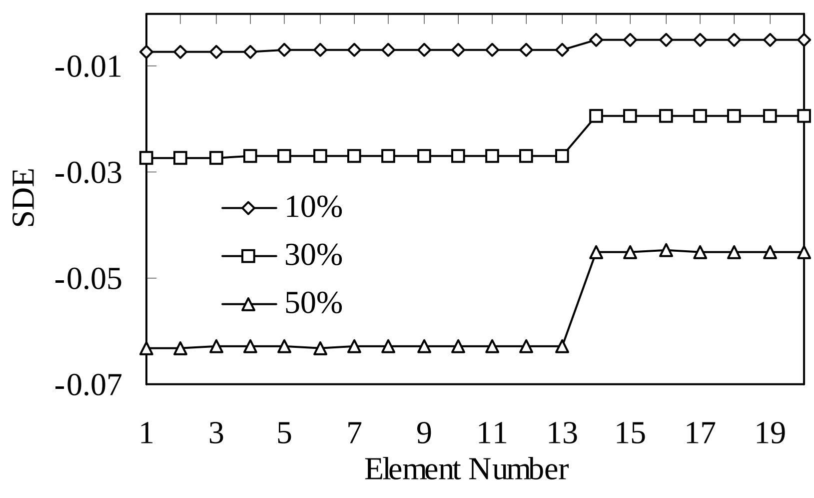


Fig. 7 The SDE for Different Severity