Experimental and Analytical Study on Visco-elasto-plastic Characteristics of PTFE-coated Glass Fiber Fabric under Cyclic Loadings

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ABSTRACT
The paper discusses the validity of the constitutive equations, previously proposed by the present authors, for the visco-elasto-plastic behaviors of fabric membranes by comparing the simulated behaviors with the experimental results recently performed also by the present authors. The constitutive equations are formulated based on Fabric Lattice Model where important material elements in the fabric material are replaced into intrinsic bar elements with time dependent behaviors as well as material non-linearities for which material constants are assumed to be as much compatible with measured results in experiments. The experiments are performed under two conditions; one is the relaxation test under a constant bi-axial strains with initial bi-axial tensions and the other one is the cyclic test under re-tension by five times after each relaxation in each re-tensioning process. The experimental results are compared with the simulated behaviors and the comparison shows a fair agreement.

1. Introduction
The stress-strain relationships of Poly-Tetra-Fluoro-Ethylene coated (abbreviated as PTFE-coated) glass fiber fabric are of high non-linearity accompanied by viscosity. Therefore, initial tensions of such membrane structures are known to decrease with years passing. The loss of initial tensions is mainly due to relaxation. In actual works in site, several hours or days are spent in the process for introducing initial tensions. During the process for initial tensioning, re-tensioning is much effective and works well against relaxation. However the initial tensions will still release to some extent after several years. The studies on the phenomenon of relaxation have been performed, especially by Nishikawa et al.[1] and Minami et al.[2]. Also, the present authors have recently performed experiments to know the effects of re-tensioning against the stress release after long periods. The simulated behaviors based on Fabric Lattice Model[3,4,5] to be compared with the results of experiments are based the constitutive equations including material non-linearities and viscosity. The viscosity is formulated by using a Voigt model with six-parameter for each bar element of which constants are characterized to match experimental results. The previous simulation based on four-parameter Voigt model [5,6] could trace the behaviors of relaxation only during four weeks, accordingly, the present study aims at a more precise simulation effective for the duration of years.

2. Fabric Lattice Model

The present study is based on the Fabric Lattice Model[3] shown in Fig.1, originally proposed for static simulation in the previous research[4], for representing the behaviors of the PTFE-coated glass fiber fabric to consider the material non-linearities and viscosity. A brief description is reviewed for simple explanation.

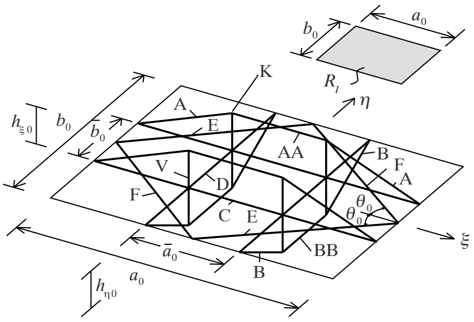

Figure 1

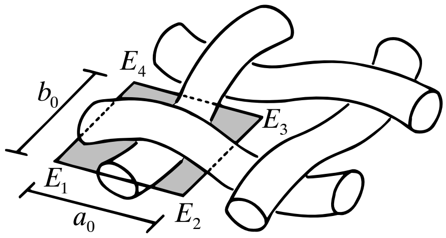

Figure 2

The trapezoidal model shown in Fig. 1 is adopted for representing the fabric membranes. The part brimmed by $E_1 - E_2 - E_3 - E_4$ in Fig. 2 is a separated portion from a membrane sheet and is transformed to the trapezoidal form, called here Fabric Lattice Model, composed of many truss bars as given in Fig. 1. In the model, (1) one warp is replaced by two rows of straight members, A, AA and A and one weft is also replaced by another rows of members, B, BB and B. The members for warps and wefts are assumed to be active only against tension because the yarns remain still bent at the beginning of loading and accordingly they seem not to be active against compression even at low stress level. (2) Coating materials are replaced by three parts. (2-1) In the first way, coatings covering front and back surfaces are represented by the parallel truss elements, C in the warp direction and D in the weft direction, and by the diagonal members, E and F. These members are assumed to be active against both of tension and compression, because the coating materials presumably work under low stress while warps and wefts remain almost inactive under low stresses. (2-2) In the second way, the coating permeated into and mingled by yarns in the middle of thickness is replaced by a sheet element R_I. This element is assumed to work only against shear deformation since the element is separated by yarns from one another. (2-3) In the third way, the coating materials pushed by warps and wefts are replaced by four vertical struts, V, which are assumed to act against compression after some amount of looseness is diminished. The hysteresis rules for the members in the fabric lattice model under static loading are omitted here and referred to the details in the previous study [4,5,7].

2.2 Formulation of Stress-Strain Relationships for Members with Viscosity

For the viscous behavior, six-parameter model illustrated in Fig. 3 is applied to each member for the fabric lattice model. The six-parameter model is composed of a Maxwell element g and a Voigt element i. The parameters E_g and E_i are the extensional stiffnesses of the Maxwell and the Voigt elements, respectively. C_g and

C_i are the compliances. The parameters η_g and η_i are the viscous coefficients.

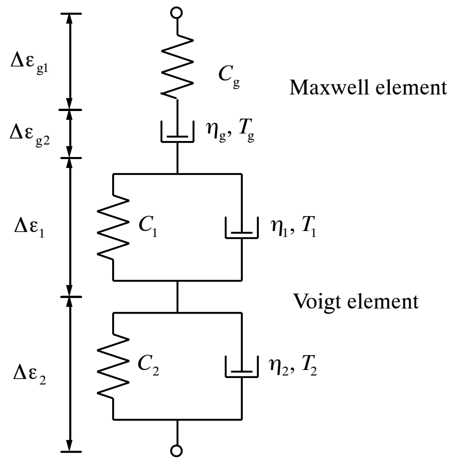

Figure 3 Six-parameter Voigt model

Among these parameters the following equations hold.

$$\begin{aligned} C_g &= 1/E_g, \quad & C_i &= 1/E_i \\ T_g &= C_g \cdot \eta_g, \quad & T_i &= C_i \cdot \eta_i \end{aligned} \tag{1}$$

where T_g and T_i are the relaxation time and the retardation time.

Based on an assumption that the static strains and the viscous strains are independent of each other, the total incremental strain $\Delta\varepsilon$ for the element in Fig. 3 is represented by the sum of the following three strains of $\Delta\varepsilon_{g1}$, $\Delta\varepsilon_{g2}$ and $\Delta\varepsilon_i$.

$$\Delta\varepsilon = \Delta\varepsilon_{g1} + \Delta\varepsilon_{g2} + \sum_{i}^{2} \Delta\varepsilon_i \tag{2}$$

where

$\Delta\varepsilon_{g1}$: elasto-plastic incremental strain of Maxwell element

$\Delta\varepsilon_{g2}$: viscous incremental strain of Maxwell element

$\Delta\varepsilon_i$: visco-elasto-plastic incremental strain of Voigt element

The three incremental strains can be obtained as Eq. (5) by assuming that the applied stress σ varies linearly from $\sigma(t_j)$ to $\sigma(t_{j+1})$ during the time increment Δt from t_j to t_{j+1}.

$$\sigma(t) = \sigma(t_j) + \frac{\Delta\sigma}{\Delta t}(t - t_j) \tag{3}$$

$$\Delta\sigma = \sigma(t_{j+1}) - \sigma(t_j),\ \Delta t = t_{j+1} - t_{j+1} \tag{4}$$

$$\left.\begin{aligned}
\Delta\varepsilon_{g1} &= C_g \cdot \Delta\sigma \\
\Delta\varepsilon_{g2} &= \frac{C_g}{T_g}\sigma(t_j)\cdot\Delta t + \frac{\Delta t}{2T_g}C_g\Delta\sigma \\
\Delta\varepsilon_i &= (1 - \frac{T_i}{\Delta t} + \frac{T_i}{\Delta t}e^{-\frac{\Delta t}{T_i}})C_i \cdot \Delta\sigma + (1 - e^{-\frac{\Delta t}{T_i}})C_i\{\sigma(t_j) - \frac{\varepsilon_i(t_j)}{C_i}\}
\end{aligned}\right\} \tag{5}$$

Substitution of Eq.(3) into Eq.(2) gives the expression for the stress-strain relationship between $\Delta\sigma$ and $\Delta\varepsilon$, leading to the following,

$$\Delta\sigma = E_T \cdot \Delta\varepsilon + f \tag{6}$$

$$\begin{aligned}
E_T &= \left[C_g + \frac{C_g}{2T_g}\Delta t + \sum_i^2 (1 - e^{-\frac{\Delta t}{T_i}})C_i(1 - \frac{T_i}{\Delta t} + \frac{T_i}{\Delta t}e^{-\frac{\Delta t}{T_i}})C_i\}\right]^{-1} \\
f &= -E_T\left[\frac{\Delta t}{T_g}C_g \cdot \sigma(t_j) + \sum_i^2 (1 - e^{-\frac{\Delta t}{T_i}})C_i\{\sigma(t_j) - \frac{\varepsilon_i(t_j)}{C_i}\}\right]
\end{aligned} \tag{7}$$

where E_T and f are the tangent stiffness and residual stress partly due to static and partly due to viscous characteristics.

3. Derivation of Constitutive Equations for the Fabric Lattice Model

3.1 Deformations

The constitutive equations for the fabric lattice model are derived on the basis of the constitutive equation for each member in the model. First, the geometries are defined for the element $E_1 - E_2 - E_3 - E_4$. The size is a_0 by b_0, and the length and sectional area of the member with a subscript K are denoted by ℓ_{0K} and A_{0K}, respectively. Second, the geometries are necessary to be defined for the deformed element. The size changes to a by b due to the strains in the warp and weft directions as well as due to the angle distortion by the shear strain. These three strains are denoted by ε_ξ, ε_η and γ, where ξ and η are a set of coordinates in the warp and weft directions. The stresses due to the strains of ε_ξ, ε_η and γ are defined by N_ξ, N_η and $N_{\xi\eta}$ in this order. Third, the strains induce the strains in each member in the model and the strain of the member with a subscript K is described by ε_K. The strain ε_K can be expressed by using the geometrical relationship between the strains of ε_ξ, ε_η and γ and the crimp interchanges in the warp and weft directions, however the details and derivation for the relationships are referred to the study[4].

3.2 Incremental Stresses Strain Relationships for Members

For the formulation of incremental stress strain relationships, the incremental strains are first defined as $\Delta\varepsilon_\xi$, $\Delta\varepsilon_\eta$ and $\Delta\gamma$ for the trapezoidal fabric lattice element. The increments change the member length of each member with a subscript K from ℓ_K to $\overline{\ell}_K$. Accordingly the incremental strain of each member is given as follows.

$$\Delta\varepsilon_K = \frac{(\overline{\ell}_K - \ell_K)}{\ell_{0K}} \tag{8}$$

The stress $\overline{\sigma}_K$ of the member after the strain increments can be expressed by using the tangent stiffness E_{TK} on the basis of Eq. (6).

$$\overline{\sigma}_K = E_{TK} \cdot \Delta\varepsilon_K + (\sigma_K + f_K) \tag{9}$$

where f_K is the stress due to the strain of ε_K and f_K is the stress increase due to the viscosity.

In the same way the shear stress $\overline{S}$ for the sheet element R_I can be defined as follows.

$$\overline{S} = k_T \cdot \Delta\gamma + (S + f_S) \tag{10}$$

where k_T is the tangent shear stiffness of the shear element and f_S is the stress increase due to viscosity.

3.2 Constitutive equations for the fabric lattice model

The virtual strain energy stored in the trapezoidal element is equal to the sum of the virtual strain energy stored in each member. Accordingly, the following equation holds.

$$a \cdot b[\delta(\Delta\varepsilon_\xi), \delta(\Delta\varepsilon_\eta), \delta(\Delta\gamma)]\begin{Bmatrix} \overline{N}_\xi \\ \overline{N}_\eta \\ \overline{N}_{\xi\eta} \end{Bmatrix} = \sum_{K=1}^{N} \delta(\Delta\varepsilon_K) \cdot \overline{\sigma}_K A_{0K} \ell_{0K} + \delta(\Delta\gamma) \cdot \overline{S} \cdot a_0 \cdot b_0 \tag{11}$$

The above equation provides the incremental constitutive equation for the fabric lattice model with a consideration of the effects of viscosity as follows.

$$\begin{Bmatrix} \overline{N}_\xi \\ \overline{N}_\eta \\ \overline{N}_{\xi\eta} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_\xi \\ \Delta\varepsilon_\eta \\ \Delta\gamma \end{Bmatrix} + \begin{Bmatrix} N_\xi \\ N_\eta \\ N_{\xi\eta} \end{Bmatrix} + \begin{Bmatrix} F_\xi \\ F_\eta \\ F_{\xi\eta} \end{Bmatrix} \tag{12}$$

where N_ξ, N_η and $N_{\xi\eta}$ are the stresses due to the strains of ε_ξ, ε_η and γ, the three terms of F_ξ, F_η and $F_{\xi\eta}$ are the increase due to viscosity, and the three terms of $\overline{N}_\xi$, $\overline{N}_\eta$ and $\overline{N}_{\xi\eta}$ are the stresses after the increments in strains occur. And the matrix $[D]$ relates the incremental strains with the stresses after the increments in strains. However, their derivation process is referred to the study[4] and the details are omitted here.

4. Experiments of Fabric Membrane under Stress Relaxation

4.1 Methods of experiment

Two types of experiments are performed. One is a pure relaxation test where the bi-axial strains are kept constant during the total time for experiments, without re-tensioning, to know the stress relaxation. The other is an experiment in which re-tensioning is applied several times at the beginning. The situation of re-tensioning is shown in Fig. 6, where bi-axial stresses are raised up after some relaxation of stresses. The second test aims to know whether the re-tensioning works effectively against relaxation. The size of tested pieces is $40cm$ by $40cm$ in Fig.4 and bi-axial stresses were introduced without shear stress. The temperature in the testing room varied from 5.7 to 26.4 degrees centigrade during the test.

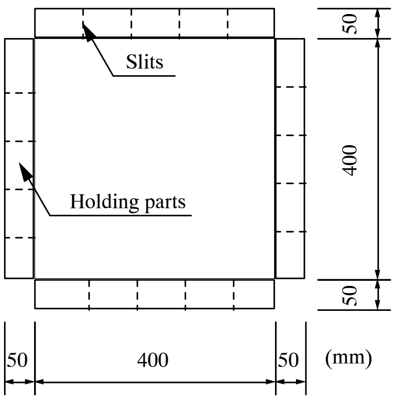

Figure 4 Experimental piece

In case of pure relaxation test, initial stresses of $N_{\xi 0} = N_{\eta 0} = 49.0 N/cm$ were applied to the tested piece which was left under constant strains for one week. ξ and η mean the warp and weft directions respectively. In the re-tensioning test, the tested piece was at first applied by the initial stresses of $N_{\xi 0} = N_{\eta 0} = 49.0 N/cm$, and was then followed by re-tensioning to the original initial stresses by four times after each six hours.

4.2 Experimental results

The results are illustrated in Figs. 5 and 6, respectively for the pure relaxation and the re-tensioning relaxation. In the pure relaxation test, the stresses decreased to 24.5 and 19.6 kgf/cm after one week, corresponding to 50 and 40% of the applied initial stresses. The stress relaxation is almost same as the experiment previously performed by Minami et al.[8]. In the re-tensioning relaxation test, the stresses decreased to 41.2 and 32.3 N/cm after the same duration of one week, corresponding to 84 and 66 % of the initially applied stresses. The re-tensioning is confirmed much effective to reduce the loss of initial stresses.

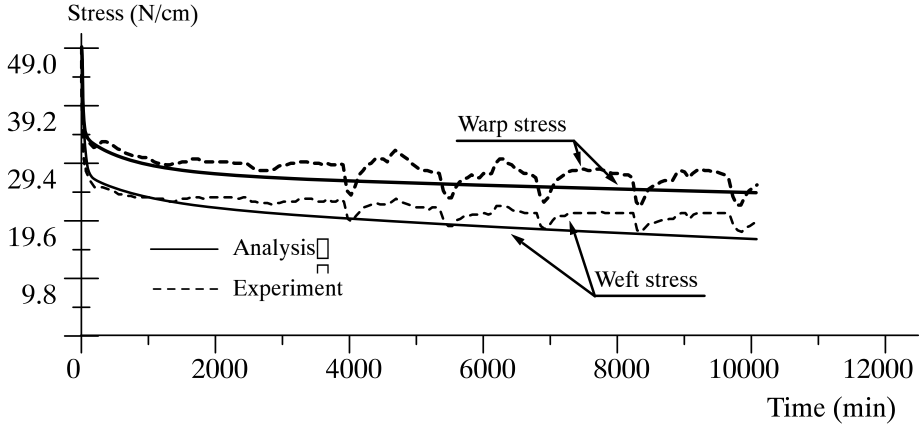

Figure 5 The results of once-loading test

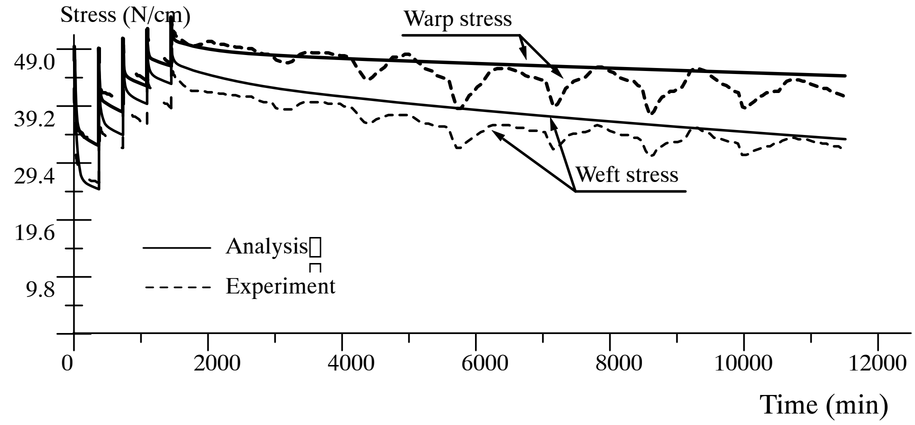

Figure 6 The results of fifth-loading test

5. Numerical Simulation based on Fabric Lattice Model

By using the constitutive equations based on the present Fabric Lattice Model, numerical simulations are performed. The parameters for static behaviors are the ones identified in the previous work[4]. The parameters for viscous behaviors are first assumed and then revised by comparing the test results with the simulated results. The comparisons are shown in Figs. 5 and 6. Although there exit some amount of discrepancies between the simulation and the experiments, it is confirmed that a good agreement is obtained.

6. Conclusion

Two types of relaxation tests are performed to investigate whether the re-tensioning process at the step to introduce initial stresses is effective to present the loss of stresses due to relaxation. The initial stresses in case of pure relaxation decreased to about 50 and 40% of the original values after one week, however, in case of re-tensioning process the stresses decreased to 84 and 66% of the initially applied stresses. Accordingly, the re-tensioning is proved much effective to reduce the loss of initial stresses.

Also, the simulated results for relaxation are given based on Fabric Lattice Model and are compared with the experimental ones. The comparison proves a good agreement between the experiments and the analysis.

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