PNEUMATIC STRUCTURES: A REVIEW OF CONCEPTS, APPLICATIONS AND ANALYTICAL METHODS

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ABSTRACT

Pneumatic structures, where pressure differentials wholly or largely ensure stability, have the potential to be considered as efficient alternatives to conventional structural forms where extreme limitations are placed upon ease & rate of construction and perhaps more importantly structural mass. Recent developments of new engineering materials such as PTFE coated nylon fabrics and other high performance structural membrane media has led to a great deal of interest in these lightweight structures. As such, air-supported and air-inflated structures have been used as emergency shelters, deployable structures for the military and to cover or span large areas.

The analysis of pneumatic structures is generally non-linear because of large displacements, potential fabric wrinkling and the orthotropic nature of the parent fabric. This has meant that various approximate numerical methods, including finite element analyses, have been employed to represent the non-linear behaviour of pneumatic structures. Mathematical techniques, based on closed form and iterative solutions of appropriate field equations (subject to boundary conditions), have also been used to model simple pneumatic forms. In addition experimental studies of full scale or model structures continue to be used in the detailed analysis and verification of membrane pneumatics. The aim of this paper is to summarise the concepts and applications of pneumatic structures and the analytical methods that have been employed in their analysis.

1.0 Introduction

Lightweight membrane structures can be divided into two separate groups based on form and structural system: suspension membrane structures and pneumatic structures.

A pneumatic structure is defined classically, as a structure in which gas or air pressure differentials control and ensure stability of form (Otto, 1968). Although air is most often used, the term has come to represent a structure in which any fluid pressure differential maintains the shape of the structure. Any applied loads are supported by an initially stressed membrane in which the initial stress is provided by internal pressure (Leonard, 1974). As such, membrane structures in general and pneumatics in particular are some of the most structurally efficient forms known. The first membrane and pneumatic structures were naturally occurring organisms and it is not surprising that inflatable structures resemble biological forms.

Structures using positive pressure as a stabilising medium have been used by mankind for thousands of years. However, it is only relatively recently that these principles have been used in the construction industry (Herzog, 1976).

Pneumatics can be sub-divided into the two main categories of air-stabilised structures and air-controlled structures (Dent, 1971).

Air-stabilised construction can be further sub-divided in to two distinct classes of structure: single wall air-supported and dual wall air-inflated structures (Kawabata & Ishii, 1994). Hybrid pneumatic forms, have been considered and may yet prove to be the most beneficial use of pneumatic technology (Dent, 1971).

Pneumatic structures have the potential to be considered when extreme limitations are placed upon structural mass and volume, rate and ease of construction and initial capital cost. Pneumatics are, however, susceptible to large displacements and membrane wrinkling under concentrated loading and have a tendency to respond severely to dynamic loads. The inherent non-linear nature of membrane and pneumatic forms also results in often complex structural behaviour under even simple loading patterns.

1.2 The Early Development of Pneumatics

Although the use of pneumatic structures in architecture is a relatively recent phenomenon, the pneumatic has been used by mankind for a number of years in a variety of applications (Herzog, 1976).

The first membrane structures were natural organisms and pneumatic technology is common in nature.

Organic cell structures made from elastic membranes are a type of pneumatic form in which the membrane is stabilised by fluids. Similarly, blood vessels are highly flexible tissues, which remain taut under the action of an internal fluid. One of the purest forms of pneumatic structure occurring in nature is the soap bubble and soap agglomeration. Soap bubbles are formed by uniform surface tensions acting upon the soap film forming minimal surface shapes (Isenberg, 1978). Any form that can be achieved by a soap film is suitable for a pneumatic. As such, soap films have been extensively employed in the empirical analysis of pneumatic forms (Otto, 1968).

Almost a third of all animals can fly (Herzog, 1976) and a majority of these use sail surfaces to achieve flight. Examples include the Dragonfly and Bat.

It is undoubtedly from observations of the various pneumatic forms in nature that man developed pneumatic concepts.

Possibly the earliest attempt to utilise pressure differentials was the simple sail, in which aerodynamic differences in air pressure cause inflation of the membrane. Early examples of closed membrane pneumatics include the inflatable animal skins used by the Romans as buoyancy bags for crossing rivers (Price, 1971).

Sail kites and box kites are a later example of an open pneumatic structure, which use pressure differentials to provide lift. Kites were developed as banners for armies in Roman. By the 15th Century, such kites had developed into reasonably complex structures that resembled mythical animals. Inflatable kites had also began to appear by this time. These relatively sophisticated closed form structures can be regarded as the forerunners of later hot air balloons.

One of the most important pneumatic structures is the pneumatic tyre, patented by R.W. Thomson (Gordon, 1988).

It is balloons and dirigibles, however, that are widely regarded as the precursors of modern pneumatic structural forms. Although the concept of balloons has existed for a number of years, it was not until the late 18th century that the hot air balloon became reality (Rolt, 1966). The first successfully manned hot air balloon, conceived by J.M. and E.J. Montgolfier, flew on 15 October 1783.

The first attempt to utilise the balloon principle for architectural structures was carried out by F.W. Lanchester (1917). In his patent for a field Hospital, Lanchester describes his plans for an air-supported building. Lanchester also investigated the potential of pneumatics for large span constructions such as aircraft hangers and sports arena.

During World War II various pneumatic structures were developed, including barrage balloons, deception devices, safety devices and inflatable boats. Other developments during the war included proposals for an air-supported roof covering a factory (Stevens, 1942) and an attempt to use pneumatics as formwork.

It was in 1946, however, that the realisation of Lanchester's idea first came to fruition. Around this time, the USA had developed large radar antennae to scan their northern frontiers. The severe climate meant that these fragile antennae had to be sheltered in some sort of non-metallic enclosure. A spherical membrane pneumatic was proposed by the Cornell Aeronautical Laboratory. Although the US government was initially sceptical about the idea, the Laboratory was awarded a contract to research the feasibility of air-supported radomes. Over a hundred of these radomes were built along the northern frontier of the USA (Bird, 1986).

The use of pneumatics in architecture has escalated dramatically since the 1950's as pneumatic warehouses, exhibition halls, stadia, shelters and greenhouses have all been constructed (Price, 1971).

2.0 Concepts and Applications of Pneumatic Structures

Of the two main categories of pneumatic structure: air-controlled and air-stabilised, the most relevant to architectural applications is the air-stabilised structure.

An air-controlled structure is a structure whose position or movement is controlled by air pressure differentials. As such this class of pneumatic is associated with mechanical and aeronautical engineering.

The form and stability of an air-stabilised structure is controlled by pressure differentials across the membrane. Internal pressure induces initial tensile stresses into the membrane, which support applied loads by a relaxation of this tensile stress.

Within this class of pneumatic, several sub-divisions can be made relating to such features as: number of membranes, type of differential pressure, magnitude of differential pressure etc (Herzog, 1976). These sub-divisions lead to two major categories of air-stabilised structure: single wall air-supported structures and air-inflated structures.

2.1 Air-Supported Structures

The most common architectural pneumatic in use is the air-supported structure, or airhouse, with an estimated 40,000 having been erected by 1980 (Happold & Dickson, 1980).

An air-supported structure consists of a single membrane held in place by a relatively low pressure differential. The air-supported structure differs from conventional structural forms in that the membrane material does not directly resist the externally applied loads. The membrane is used to contain an internal volume of air, which supports applied loads. In theory if the applied loading were uniform and equal in magnitude to the internal pressure, the membrane would merely act as a separating medium and would be free of any tensile stresses (Otto, 1968). In practice, however, surface loads are never uniform and the internal pressure must be maintained at a higher level to prevent compressive stresses in the membrane.

It was as military applications that air-supported structures first proved their worth. The first air-supported structures were the DEW-Line radomes (Bird, 1986). The outstanding performance of these structures in severe climatic conditions confirmed the practicality of pneumatic construction and commercial applications quickly evolved.

Industrial applications have proven to be one of the areas in which air-supported forms are most useful. Storage and warehousing problems have led to the construction of numerous examples of air-supported shelters. Portable and flexible air structures can be built when needed and at virtually any convenient location. Rapid expansion can also facilitate the need for an instant increase in production in factories and plants. Air-supported construction has been used to provide both temporary and permanent factory space. Air structures have also been used as shelters for construction activities in cold and extreme climates (Mangus, 1991). The air-supported covering provides an encapsulated worksite that can be heated allowing personnel to work more efficiently.

One of the most prolific uses of airhouses is as sports enclosures. Swimming pools, ice rinks, tennis courts, football fields and multi-purpose halls (Sugizaki, 1986) have all been covered.

It is in the field of Exhibition structures that significant architectural development has been made. Air structures are extremely suitable as exhibition structures due to their novel design and mobility. One of the earlier examples is the Pentadome Exhibit built for the US Army in 1958. This exhibit consisted of 5 hemispherical air-supported domes. The larger, 49m diameter, central dome was surrounded by 4 smaller domes. Other examples include the Krupp exhibition pavilion built in 1966 in Hanover. This air-supported hall, covering 3,300m^2 was built in combination with more conventional structural forms, to house conference rooms and service areas. A notable example is the US Pavilion built at EXPO ’70 in Osaka, (Geiger, 1970; Yida *et al*, 1986; Isono and Nakahara, 1986). This construction, covering almost 10,000m^2, was one of the first air-supported cable reinforced roofs to be built. A low roof profile was designed and so a diamond grid of steel cables was included to reduce the radii of the membrane and so reduce tensile stresses in the fabric material.

More recent applications include the use of pneumatic roof coverings for large stadia and arena. The largest air-supported roof covering a sports stadium is the cable-reinforced roof over the Pontiac Silverdome Stadium (Rigoni, 1977). The roof is made from Teflon coated fibreglass fabric and is reinforced with a network of 18 steel cables in order to reduce the tensile stresses in the membrane. Other examples include the Tokyo Dome and the RCA Dome in Indianapolis.

2.2 Air-Inflated Structures

An air-inflated structure consists of closed membrane sections inflated to form structural components. These structural elements resist external loads in a similar way to more conventional structural elements.

Air-inflated structures can be sub-divided into tubular structures and dual-wall, or cushion structures.

Tubular or rib structures are made up of a framework of pressurised hoses, often supporting a fabric membrane. Inflated dual-wall structures consist of two membrane walls stabilised by an internal pressure. The two walls are often connected by drop thread or diaphragms and if the drop threads are closely spaced together, the structural element is known as an “Airmat”. Although the use of air-inflated structures in architectural applications is not as common as that of air-supported forms, air-inflated structures are abundant in other areas. Non-architectural applications include, life jackets, inflatable furniture, beach balls, mattresses, inflatable boats, aircraft escape slides and car airbags.

Air-inflated tubular and dual-wall structures lend themselves to situations where ease of construction and structural mass are crucial. As such proposals have been made to use pneumatic rib structures in outer space (Leonard *et al*, 1960; Harris and Stimler, 1961).

The first attempt to use air-inflated structures in space was the passive telecommunications satellite ECHO-1, launched in 1960. This 30m diameter spherical balloon was made from metallized mylar and had a packing diameter of only 70cm. Further inflated satellites were launched as ECHO-2 in 1964 and PAGEOS-1 in 1966. Proposals have been made to use skeletal inflatables as deployable space structures to support antenna reflectors (Girard et al, 1982; Reibaldi, 1985; Authier and Hill, 1985; Bernasconi, 1986; Miura *et al*, 1986) and as solar concentrators (Grosman and Williams, 1989). Complex inflatable structures have also been proposed for Lunar and Martian outposts (Steinberg and Bulleit, 1994; Drake and Richter, 1992; Chow, 1992; Zuppero *et al*, 1994; Nowak *et al*, 1992; Sadeh *et al*, 1996a; Sadeh *et al*, 1996b; Sadeh *et al*, 1996c).

The use of air-inflatables has also been proposed for re-entry vehicles (Leonard *et al*, 1960) and aeronautical applications (Webber, 1982; McQuaile, 1981), the Pathfinder module used an airbag impact attenuation system in its landing on Mars (Rivellini, 1996).

Architecturally dual-wall and tubular structures have been employed as Radomes, roof structures, framing systems for fabric structures, inflatable tents and as mobile shelters. An interesting example of a tubular air-inflated structure is the Medical Unit, Self-contained, Transportable (MUST) shelter. The MUST shelter consists of 12 individual cylindrical tubes placed adjacent to one another to form a semi-cylindrical building. Development of air-inflated forms for the military has resulted in some interesting concepts, including a portable inflated bridge made from a cellular construction (Bulson, 1967; Bulson, 1971).

The roof over the Roman amphitheatre in Nimes, France is a dual-wall cushion construction (Schlaich *et al*, 1989). This cushion structure has been designed to cover the central part of the amphitheatre in the winter.

Some of the more innovative architectural forms of construction have been presented at Exhibitions and in particular at World Expositions. EXPO '70 produced two excellent examples of air-inflated tubular structures, the Floating Theatre Pavilion and the Fuji Group Pavilion (Nohmura, 1991).

The Floating Theatre is a hybrid construction utilising both air-inflated tubular construction and concepts associated with air-supported structures. The theatre was enclosed by a single roof membrane that was held in place by three air-inflated beams forming 23m diameter arches. The membrane roof was stabilised by a negative internal pressure, forming an anticlastic surface.

The Fuji Group Pavilion was constructed from 16 air-inflated arches. These arches were positioned adjacent to each other in a circular plan so that in the centre, the tubes formed a semi-circular shape but at either end the arches were brought closer together, making their apex higher.

3.0 Analytical Methods

The analysis of a pneumatic membrane structure may be carried out using either mathematical techniques, numerical methods or experimentally.

Mathematical and numerical techniques have many similarities in their formulation and method of solution. Numerical procedures must be based upon a mathematical relationship and many closed form solutions rely upon an iterative solution technique. It is in the representation of the structural model where the two methods differ.

Extensive research has been undertaken into the study of linear shell and membrane analysis problems (Leonard, 1974; Jenkins and Leonard, 1991; Jenkins, 1996).

Although a comprehensive understanding of the complex behaviour of pneumatics has yet to be achieved, certain particular forms can be modelled using mathematical techniques. Spherical, cylindrical and many axisymmetric pneumatic forms have been analysed mathematically.

The fundamental relationship between membrane stress and shape in a pneumatic structure for simple forms can be found by resolving forces normal to the plane of the membrane. This leads to a formula relating the membrane tensions to the principle radii of curvature for a given internal pressure:

$$\frac{N_1}{r_1} + \frac{N_2}{r_2} = P$$

where N_1 and N_2 are the membrane tensions, r_1 and r_2, the principle radii of curvature and P, the internal pressure.

For a sphere, where the principle radii of curvature are equal, the stress at every point on the surface and in every direction is the same and equal to:

$$N_1 = N_2 = \frac{P.r}{2}$$

The radial tensions, N_1, in a cylindrical form can also be found as $r_1=r$ and $r_2=\infty$. Hence:

$$N_1 = P.r$$

The longitudinal tensions, N_2, are indeterminate and depend upon end conditions.

These simple linear relationships have been used in the analysis of the flexural response of inflated circular cylindrical beams. The structural performance of inflated cantilever beams under bending, torsion and buckling have all been undertaken (Topping 1964; Bulson, 1973; Webber, 1982). Linear shell analyses have been used to predict the buckling behaviour of inflated members (Leonard *et al*, 1960) in which an expression for the collapse load of a cantilever beam is derived. Comer and Levy (1963) investigated the behaviour of cylindrical beams between incipient wrinkling and final collapse using a method analogous to conventional beam theory. They derived an expression for the deflections of the cantilever in terms of two dimensionless variables. Main *et al.* (1992) reformulated this model using stress resultants in order to make it more applicable to fabric structures. The same authors continued this work in two further papers (Main *et al*, 1994; Main *et al*, 1995) in which a revised bending model was formulated to take into account the biaxial stress state within the structure. Non-linear methods have also been employed to model the stiffness of inflated cantilever beams (Douglas, 1969).

A great deal of work has been carried out regarding the non-linear response of membrane materials, based on a variety of different formulations of the initial boundary value problem.

A significant amount of research into the response and analysis of membranes develops from work carried out by Adkins and Rivlin (1952) and the large deformation theory presented by Green and Adkins (1970).

Trostel (1962) describes a number of methods for the analysis of membranes including a closed form solution to the inflation of a plane circular membrane, first outlined by Adkins and Rivlin (1952). Foster (1967) also used a closed solution to find the displacements of a neo-Hookean circular membrane. In references from Hart-Smith and Crisp (1967) and Yang and Feng (1970), numerical solutions are given for this problem. Tielking and Feng (1974) analysed a plane circular membrane using a potential energy minimisation technique. The inflation of spherical, toroidal and cylindrical membranes have also received some attention. Green and Adkins (1970) considered these problems. A Eulerian formulation has been used to analyse spherical membranes (Vishwanath and Glockner, 1972) and the effects of axisymmetric loads on inflated spherical membranes (Vishwanath and Glockner, 1973). Chen and Healey (1991) studied the problem of non-spherical axisymmetric equilibria of an initially spherical membrane.

The finite inflation of a toroidal membrane has been investigated in papers by Kydoniefs (1967), Kydoniefs and Spencer (1967) and Feng (1976).

Kydoniefs and Spencer (1969) subsequently gave an exact solution to the differential equations governing the deformation of a cylindrical membrane. The equilibrium of an inflated cylindrical membrane in contact with two rigid cylindrical surfaces has been investigated by Callegari and Keller (1974). Matsikoudi-Iliopopoulou (1987) considered the axisymmetric deformation with torsion of a cylindrical membrane of a Mooney-Rivlin material. More recently, Hart and Shi (1991) investigated the problem of two inflated cylindrical membranes joined longitudinal at a cross-section.

The analysis of inflatable shells of revolution has been carried out for the three stages in the behaviour of an inflatable shell: unfolding phase, pressurisation phase and in-service phase (Leonard, 1967a; Leonard, 1967b; Leonard, 1967c).

Finite element theory has been extensively used in the analysis of numerous pneumatic forms. A number of references outline the numerical formulation and solution of non-linear elasticity problems (Oden, 1967; Poskitt, 1967; Mallet and Marcel, 1968). In addition, several finite elements have been developed that may be used, with an appropriate non-linear solution technique, in the analysis of pneumatics (Cook, 1989; Zienkiewicz and Taylor, 1988, Levy and Spillers, 1994).

Oden and Sato (1967) and Oden and Kubitza (1967) used flat triangular elements and a Mooney-Rivlin material model in their analysis. Li and Leonard (1973) used a non-linear curved quadrilateral finite element for analysis of a spherical inflatable membrane. Leonard and Verma (1976) developed a quadrilateral curved element based on a polynomial surface patching technique specialised for a Mooney-Rivlin material. This element was also used in the study of the behaviour of cylindrical and initially flat, cable-reinforced membranes (Leonard and Verma, 1978). Haug and Powell (1986) formulated a quadrilateral finite element for the analysis of a series of membrane structures including minimal surfaces and inflated forms. The analysis of a number of membrane structures, using both membrane and cable finite elements, are reported by Magara and Okamura (1987) and Okamura and Magara (1987). Geometrically and materially non-linear orthotropic hyperelastic membranes, representing biological tissues, have been studied using a finite element model (Kyriacou *et al*, 1996).

The analysis of pneumatic and membrane structures has also been carried out using non-linear finite element formulations with some success (Williams, 1980; Schwenkel, 1983; Han and Olson, 1986; Ishii, 1986; Nishimura *et al*, 1986; Okamura and Magara, 1987; Han, 1989; Gosling and Lewis, 1996a; Gosling and Lewis, 1996b; Berry and Yang, 1996).

Kawabata and Ishii (1994) used a non-linear finite element formulation to analyse a number of air-inflated beam structures. The analysis considered the orthotropic nature of the fabric and potential wrinkling. Gosling and Riches (1997) used a mixed or hybrid approach to investigate this problem. A geometrically non-linear triangular finite element was used with the dynamic relaxation algorithm to find the displacements of a number of laterally loaded inflated cantilever beams.

The dynamic relaxation algorithm is one of a number of solution techniques available for non-linear membrane analysis. This iterative technique, first presented by Day (1965) uses fictitious values of mass and a forced pseudo vibration of the structure to find the equilibrium configuration. Barnes (1980) outlined a number of appropriate non-linear solution methods for the analysis of pneumatic structures including the Dynamic Relaxation algorithm. The methods reviewed were classified into iterative, incremental and minimisation and relaxation methods.

Incremental and iterative solution methods are the most widely used. These two procedures are easily applied to matrix formulations of the overall stiffness of the structure. Minimisation and relaxation techniques are formulated as a vector method of individual components, or elements, of the structure. The most common and stable iterative analysis solution method is the Newton-Raphson method. An iterative solution method is concerned with locating a single point on a particular curve. If the entire curve is required an iterative process can be repeated after convergence, at each load required. If, however, the load is increased in each computational cycle, the solution is known as an incremental method (Cook, 1989). This solution method is useful for structures that are subject to both material and geometric non-linearity with a path dependant solution.

Experimental studies are invaluable in the detailed analysis and verification of pneumatic forms. Experiments may be used in the verification of structural models, to determine material properties and to provide detailed information about the structural form involved. Otto (1962) gave various general guidelines for the modelling of pneumatic forms. Various experiments have been carried out in order to determine fabric material properties including, stiffness and breaking strength (Topping, 1961; Green and Adkins, 1970; Fritzsche, 1967; Ansell and Harris, 1980; Reinhardt, 1976; Shimamura and Takeuchi, 1986; Chen *et al*, 1995). British and European standards also give details relating to the testing of coated fabrics although no guidelines are presented for the determination of modulii of elasticity (BS3424, 1982; EN ISO 2231).

The low rigidity and elastic modulus of membrane structures has meant that non-invasive, non contact experimental methods are necessary. The grid and Moire methods, using a ruled grating surface or grid, have been used with some success (Durelli *et al,* 1967; Durelli and Chen, 1973). The grid method uses a series of orthogonal lines marked onto the undeformed surface of the structure, and through direct photographic measurement of this grid, displacement/strain information is calculated. The Moire method uses a second grid pattern to obtain this information.

Numerous references have used experimental methods to validate both mathematical and numerical methods (Foster, 1967; Webber 1982; Main *et al*, 1994; Kawabata and Ishii, 1994 etc.).

Experimental techniques are used extensively to reveal structural characteristics under external loads and in particular dynamic wind loading. Wind tunnel tests on spherical air-supported structures were carried out at Cornell Aeronautical Laboratories from 1946 (Bird, 1986). Wind tunnels have also been used in the analysis of an inflated tent structure (Ross, 1969) and on radomes (Newman and Goland 1982). Full scale tests have been carried out on some innovative inflated devices detailed by Bulson (1967). Large scale vibration experiments were performed to study the characteristics of a cable-reinforced air-supported structure (Takeda *et al*, 1986). Similar experiments have been performed upon various air-supported forms (Nakayama *et al* 1986; Fukao *et al* (1986); Kawamura and Kiuchi, 1986; Kaseem and Novak, 1991).

4.0 Potential Research

The use of pneumatics in architecture and mechanics has increased dramatically over the last 50 years. In this time a great deal of progress has been achieved both in the practical use of pneumatics and in the analysis of pneumatic forms. The theories regarding large non-linear deformations and membrane theory are now well established. The mathematical analysis of axisymmetric deformations has received a great deal of interest and as such is now well understood. Likewise, much research has been conducted into numerical techniques. However, an accurate and fully versatile solution method is still to be found.

It is likely that research will begin to centre upon specific problems that have been previously encountered.

The modelling of areas of potential wrinkling in non-compression fabric materials has been investigated and tension field theories have been formulated. However, the tension field theory does not, give any information regarding the area of wrinkling.

Additional pneumatic forms continue to emerge and research into the development of a novel structure that embraces the advantages of lightweight tension structures and the versatility of skeletal air-inflated structures is in progress.

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